National Technical University of Athens

Cartography Laboratory

Choosing a Suitable Projection for Navigation in the Arctic

ZATTA "Carta Delle Due Regione Polare." Venice. c. 1778

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1 Introduction

As navigation in the Arctic region becomes a reality due to the progressive melting of the Polar Ice Cap, there is a need for cartographers and ECDIS manufacturers to develop and incorporate in their systems software for suitable polar projections. Because of the large distortions in areas and angles that the Mercator and other projections cause, the selection of a suitable projection for navigation in the Arctic must be revisited. More specifically for the projections used in ECDIS and in order to ensure commonality of display and simplify the task of both the user and the Type Approval authorities, there is a need for standardization of the display through the use of suitable projections for the Arctic and the sub Arctic areas. The study area for this project extends from 50° to 90° [lat] although the Arctic Region itself is considered to be north of the 68th parallel (fig.1). This is due to the fact that it was considered necessary to address the issue of the selection of suitable projections for the sub-Arctic Region.

The factor of high latitudes directs nautical cartographers to consider the Arctic Region as a navigation area with special conditions. The navigators' traditional way of observing a charted sea area is using the Mercator projection with meridians perpendicular to the parallels. This pattern of straight orthogonal graticule lines – besides the eventual contribution to navigation - has the advantage that allows for direct conclusions on the relative geographical location between any two points on the chart.. In the polar areas the situation is different. Here the meridians are converging towards the poles, which are the center for the concentric latitude circles. The extreme convergence of the meridians makes it inappropriate or impossible to determine directions on these charts. A compass line will be curved and will differ considerably from a great circle, even at short distances. Visual bearings cannot, or only with inadequate accuracy, represent a compass line on the chart. This means that the Mercator chart becomes useless at higher latitudes, as the compass lines become less usable. The visibility in polar areas can be considerable and visual bearings of many nautical miles can be experienced. Using such a bearing on a nautical chart can give diverging results. Another disadvantage with the Mercator chart is the North – South variation in scale, which results in considerable errors in distance measurements and area distortion. It will be necessary for the ECDIS to use another projection suitable for the area. However, other projections than Mercator have not been included in the current standards. In addition to the above, magnetic variation at high latitudes is considerable and can reach up to 180° as the magnetic pole has a dynamic position away from the geographic pole. Paper charts should be consulted for information, bearing in mind that the accuracy of isogons information can be less than in charts covering non Arctic areas. Moreover, as both magnetic poles are in the polar regions, the horizontal intensity of the earth's magnetic field is not significant and it is weak for magnetic compasses which become useless in some areas. The phenomenon of *aurora* caused by magnetic storms can also influence a magnetic compass [ArHC2-09A INF1, 2011].

The IMO performance standards for ECDIS do not provide for specific requirements or recommendations on the employment of specific map projections. Navigation with ECDIS has simply assumed use of the Mercator projection, since this projection is applicable for the current area of ENC coverage. As a result, all ECDIS and ECS manufacturers have developed systems, particularly software, that use the Mercator projection (and in some cases other projections) and this is the display capability that has been given approval by the Type Approval authorities [ARHC2-08A, 2011].

The Electronic Navigational Chart (ENC) data collected by Hydrographic Offices in support of ECDIS have no dependence on specific projection giving way to the use of any projection that will be considered as suitable. It is noted that the lack of official standardization or detailed recommendations on the use of specific map projections in ECDIS does not cause any problem on its function to contribute to the computations required for safe navigation. This is due to the fact that computations in ECDIS are conducted analytically on the surface of the reference ellipsoid [WGS84] utilizing geo coordinates.

In traditional navigation methods where the Mercator and Gnomonic charts are used due to their inherent characteristics to portray Rhumb Lines (RLs)/Loxodromes, and the Great Circles (GCs)/Orhodromes with straight lines, graphical work on the surface of the projection is required. Consequently in ECDIS as well as in other navigational systems, there is no need to use a particular map projection for computations. Nevertheless the lack of standardization on the employment of map projections in ECDIS has resulted in the relaxed use of projections and their parameters in many commercial ECDIS systems, which - in some cases - cause remarkable visual distortions and misinterpretations of the reality [Pallikaris and Tsoulos, 2009].

According to ARHC the use of a projection other than the most suitable one may raise several issues/questions such as [ARHC2-08A, 2011]:

a. Issues related to radar integration with ENC
b. Display issues related to ships navigating with charts on different projections in proximity of one another
c. Use of a standard projection becomes essential if there are users who need grid directions in that on the same projection with the same standard parallels, a single grid direction will exist between any two points
d. Is there a need for specification of a plane at some parallel to reduce scale variation?
e. Is there a need for an Arctic Test Data set for manufacturers and Type Approval authorities to test ENC functionality where longitude lines and datelines converge?

1.1 Projections for the Arctic

Special consideration is given to the selection of projections for the Arctic because the projections commonly used become special cases with unique features [American Practical Navigator, 2002]:

In the case of **Cylindrical** projections in which the axis of the cylinder is parallel to the polar axis of the earth, distortion becomes excessive and the scale changes rapidly. Such projections cannot be carried to the poles. However, both the Transverse Mercator [TM] and Oblique Mercator [OM] projections may be used for large scale charts and for mapping areas that are mainly North-South in extent. This is due to the fact that distortion of distances, directions and size of areas increases rapidly away from the central meridian [TM] or the great circle [OM].

Conic projections with their axes parallel to the earth's polar axis are limited in their usefulness for polar charts because parallels of latitude extending through a full 360° of longitude appear as arcs of circles rather than full circles. This is because a cone, when cut along an element and flattened, does not extend through a full 360° without stretching or resuming its former conical shape. The usefulness of such projections is also limited by the fact that in some of them the pole appears as an arc of a circle instead of a point. However, by using a parallel very near the pole as the higher standard parallel, a conic projection with two standard parallels can be made. This requires little stretching to complete the circles of the parallels and eliminate that of the pole. Such a projection, called a modified Lambert conformal projection, is useful for polar charts. It is particularly familiar to those accustomed to using the ordinary Lambert conformal charts in lower latitudes.

Azimuthal projections are in their simplest form when tangent at a pole. This is because the meridians are straight lines intersecting at the pole, and parallels are concentric circles with their common center at the pole. Within a few degrees of latitude off the pole they all look similar; however, as the distance becomes greater, the spacing of the parallels becomes distinctive in each projection (fig. 2). In the polar azimuthal equidistant it is uniform; in the polar stereographic it increases with distance from the pole until the equator is shown at a distance from the pole equal to twice the length of the radius of the earth; in the polar gnomonic the increase is considerably greater, becoming infinity at the equator; in the polar orthographic it decreases with distance from the pole. All of these but the last are used for polar charts.
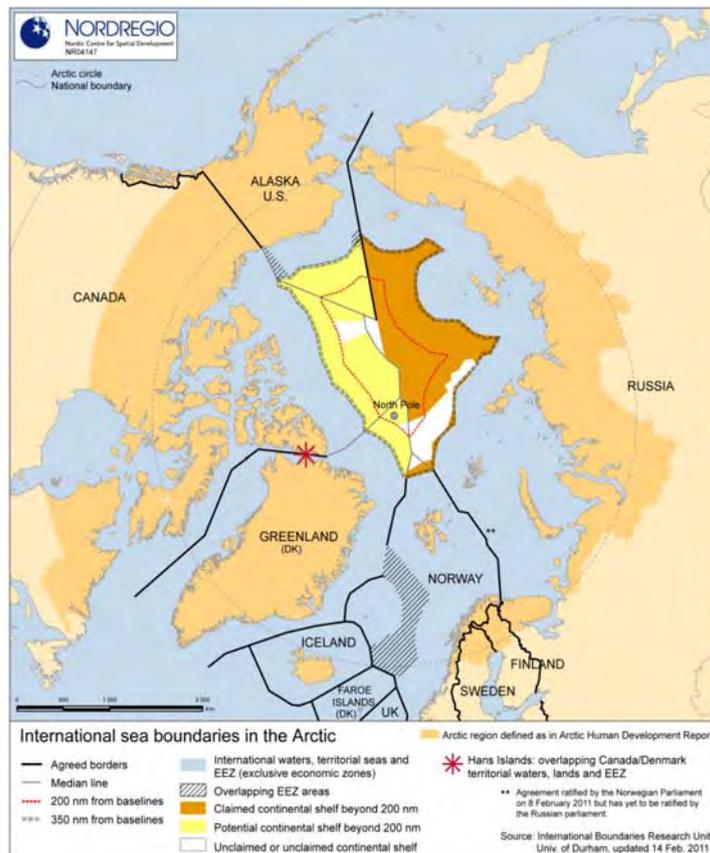
Figure 1. Arctic Region
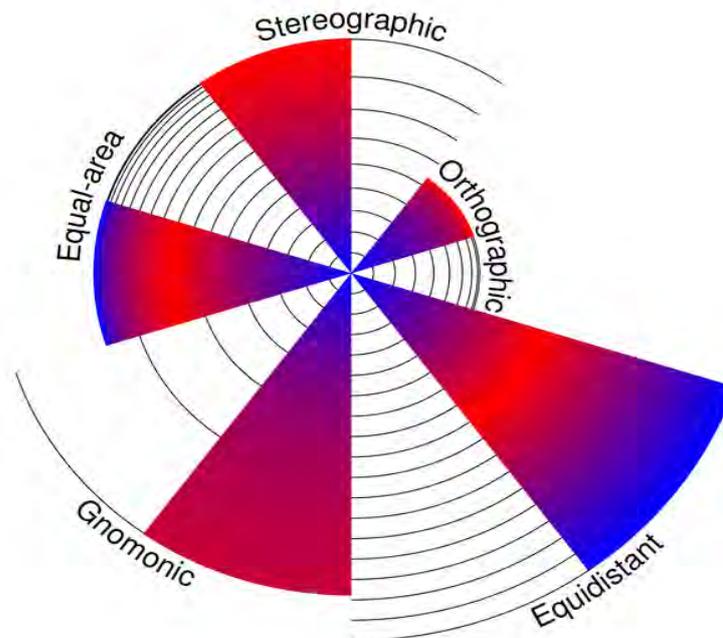
Figure 2. Compared polar aspects of selected Azimuthal projections at identical scale, parallels spaced 10° apart (equatorial zone in red, polar caps blue).

Based on the above the principal considerations in the choice of a suitable projection for polar navigation are:

a. Conformality: When the projection represents angles correctly, the navigator can plot directly on the chart
b. Great circle representation: Because great circles are more useful than rhumb lines at high latitudes, the projection should display great circles as straight lines
c. Scale variation: The projection should have – if possible - a constant scale over the entire chart
d. Meridian representation: The projection should show straight meridians to facilitate plotting and grid navigation and – if possible – straight parallels perpendicular to meridians
e. Limits: Wide limits of application reduce the number of projections needed to a minimum.

The projections commonly used for polar charts are: the modified Lambert Conformal, the Gnomonic, the Stereographic and the Azimuthal Equidistant (Pearson, 1990). These projections are similar near the pole. They are essentially conformal, and a great circle on each is nearly a straight line. As the distance from the pole increases, however, the distinctive features of each projection become apparent:

a. The modified Lambert conformal projection is conformal over its entire extent. The amount of scale distortion is comparatively small if it is carried only to about 25° or 30° from the pole. Beyond this, the distortion increases rapidly. A great circle is very nearly a straight line anywhere on the chart. Distances and directions can be measured directly on the chart in the same manner as on a Lambert conformal chart. However, because this projection is not strictly conformal, and on it great circles are not exactly represented by straight lines, it is not suited for highly accurate work.
b. The Polar Gnomonic projection is the one polar projection on which great circles are exactly straight lines. However, a complete hemisphere cannot be represented upon a plane because the radius of 90° from the center would become infinity.
c. The Polar Stereographic projection is conformal over its entire extent and a straight line closely approximates a great circle. The scale distortion is not excessive for a considerable distance from the pole, but it is greater than that of the modified Lambert conformal projection.
d. The Polar Azimuthal Equidistant projection is useful for showing a large area such as a hemisphere because there is no expansion along the meridians. However, the projection is neither conformal nor equivalent and distances cannot be measured accurately in any but a north-south direction. Great circles other than the meridians differ somewhat from straight lines. The equator is a circle centered at the pole.
e. The two projections most commonly used for polar charts in traditional navigation are the modified Lambert Conformal and the Polar Stereographic. When a directional gyro is used as a directional reference, the track of the craft is approximately a great circle. A desirable chart is one on which a great circle is represented as a straight line with a constant scale and with angles correctly represented. These requirements are not met entirely by any single projection, but they are approximated by the modified Lambert Conformal, the Polar Stereographic and the Azimuthal Polar Equidistant. The scale is more nearly constant on the Polar Equidistant, but the projection is not strictly conformal. The Polar Stereographic is conformal, and its maximum scale variation can be reduced by using a plane which intersects the earth at some parallel intermediate between the pole and the lowest parallel. The portion within this standard parallel is compressed and that portion outside is expanded.

1.2   Analytical approach for the selection of a suitable arctic projection

Based on the above bibliographical information one could proceed by simply choosing a projection among those suggested. We opted for another - namely the analytical - approach due to the following reasons:

  a. The selection of the suitable projection should be based on a number of criteria, which are quite relevant to the angular, area and distance distortions inherent in each projection
  b. These distortions vary with the latitude and their variance is not constant. The analytical approach is the only way to quantify each kind of distortion and to determine the area where the set criteria are satisfied
  c. Besides the above and particularly for the Arctic Region there is a need to identify the limit between Arctic and Sub-Arctic areas in order to propose suitable projection(s) for the latter.

The selection of a suitable projection for use in the Arctic and sub-Arctic Region depends on mission requirements. These requirements establish the relative importance of various features. For a relatively small area, any of several projections is suitable. For a large area, however, the choice is more difficult. If grid directions are to be used – as in navigation - it is important that all units in related operations use charts on the same projection, with the same standard parallels, so that a single grid direction exists between any two points.

The desired characteristics of the map projections for nautical charts and ECDIS/ECSs that have been taken into account in this study are:

  a. The control of the amount of distortion within acceptable limits (minimization and improvement of the distribution of distortion).
  b. The shape of Great Circles (GCs) and Rhumb Lines/Loxodromes
  c. The shape of the Graticule lines

1.2.1   The criterion of minimization and distribution of distortions

The values of all kinds of distortions (linear, area and angular) depend on the scale distortion on the principal directions of the projection [Maling, 1973]. Since for small values of angular distortion, the scale distortion on the principal directions is about the same, it is sufficient to consider the values of area distortion [Delmell, 2001]. For these reasons in the analytical approach followed for the choice of suitable projection(s) in the Arctic/sub-Arctic Region for nautical charts and ECDIS/ECSs, the maximum desired tolerances are set only for the angular and area distortions. These tolerances – or acceptable limits of distortion – can take the following values:

  a. 8°- 10° for angular distortion and to 8% -10% for the area distortion, in order ensure that no serious visual distortion is generated
  b. 6°- 8° for angular distortion and to 6% - 8% for the area distortion, in order ensure that no visual distortion is generated [Bugayevskiy and Snyder, 1995]
  c. Up to 1° for angular distortion and up to 2% for the area distortion, in order to allow for cartometric use of the map/chart [Pallikaris, 2010]

It is pointed out that the tolerance values depend always on the size of the area covered, the corresponding scale and the intended use of the map/chart. According to the existing bibliography this criterion can be satisfied by a Cylindrical, Conical or Azimuthal projection. These projections are studied analytically in order to achieve the maximum possible reduction of area and angular distortions and better compromise the conflicting requirements for the desired shape of Great Circles and Graticule lines.

1.2.2 The criterion of the shape of Great Circles

For better portrayal of the reality the lines showing Great Circles have to be shorter than the corresponding Rhumb Lines/Loxodromes. In addition Great Circles should preferably be straight lines and when curved they should bend towards the equator and Rhumb Lines/Loxodromes towards the poles.

This criterion can be satisfied by the Conic and Azimuthal family of projections on which Great Circles are shown with shorter lines than Rhumb Lines/Loxodromes. In addition, these projections have the basic advantage of reducing significantly the amount of angular and area distortions of the Gnomonic projection used in traditional navigation.

Figure 3. Portrayal of the false impression given in ECDIS when graticule lines are not shown

1.2.3 The criterion of Meridian representation

In navigational systems it is very useful to ensure that meridians are portrayed as straight lines and intersected orthogonally with parallels. Besides the eventual contribution to navigation (to facilitate plotting and grid navigation) it allows for direct conclusions concerning the relative geographic location of any point from the overall visual appearance of the area portrayed. On the other hand such conclusions should not depend on the portrayal of the meridians and parallels, which are not always displayed on ECDIS. Therefore there is a risk in creating false impressions, as in the example shown in fig. 3.

The family of cylindrical projections on which meridians and parallels are shown as straight lines intersected orthogonally - as on the Mercator projection commonly used in traditional nautical charts - satisfies this requirement. In this approach, the compensation cost of ensuring the desired shape of meridians and parallels, is that the criterion for the shape of Great Circles cannot be satisfied (Pallikaris and Tsoulos, 2010).

Realizing that the abovementioned criteria for the selection of suitable map projections cannot be satisfied simultaneously, the evaluation of the map projections studied is carried out separately for each criterion in the order they appear in this document. The outcome of the evaluation of the projections with respect to the first criterion is taken into account for the second and the third.

1.2.4 Scale Variation

Since no map projection maintains true scale throughout, it is important to determine the extent to which it varies on a map/chart. On a world map distortion is evident to an eye familiar with maps after noting the extent to which landmasses are improperly sized or out of shape, and the extent to which meridians and parallels do not intersect at right angles or are not spaced uniformly along a given meridian or given parallel. On maps of countries or even of continents, distortion may not be evident to the eye, but it becomes apparent upon careful measurement and analysis.
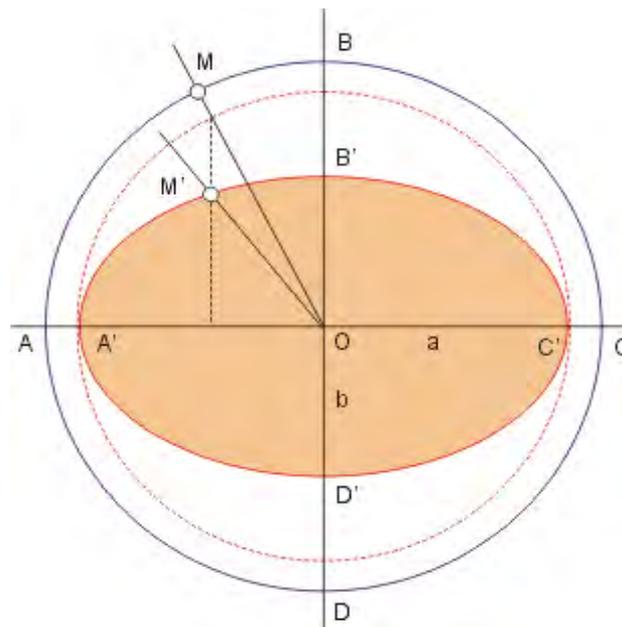

Figure 4. Tissot's indicatrix

1.2.5 Tissot's Indicatrix

In 1859 and 1881, Nicolas Auguste Tissot published a classic analysis of the distortion which occurs on a map projection [Snyder, 1987; Maling, 1973]. The intersection of any two lines on the Earth is represented on the flat map with an intersection at the same or a different angle. At almost every point on the Earth, there is a right angle intersection of two lines in some direction (not necessarily a meridian and a parallel), which are also shown at right angles on the map (principal directions). All the other intersections at that point on the Earth will not intersect at the same angle on the map, unless the map is conformal, at least at that point. The greatest deviation from the correct angle is called ω, the maximum angular deformation. For a conformal map, ω is zero. (In some texts, 2ω is used rather than ω).

Tissot showed this relationship graphically with a special ellipse of distortion called an indicatrix (fig. 4). An infinitely small circle on the Earth is projected as an infinitely small, but perfect, ellipse on any map projection. If the projection is conformal, the ellipse is a circle, an ellipse of zero eccentricity. Otherwise, the ellipse has a major axis (a) and minor axis (b), which are directly related to the scale distortion and to the maximum angular deformation.

Scale distortion is most often calculated as the ratio of the scale along the meridian or along the parallel at a given point to the scale at a standard point or along a standard line, which is made true to scale. These ratios are called "scale factors". The one along the meridian is denoted as h and that along the parallel, as k. The term "scale error" is frequently applied to (h-1) and (k-1). If the meridians and parallels intersect at right angles, coinciding with a and b the scale factor on any other direction at such a point will fall between h and k. Angle ω may be calculated from equation $\sin(\omega/2)=|a-b|/(a+b)$, substituting a and b by h and k. In general, however, the computation of ω is much more complicated, but is important for knowing the extent of the angular distortion throughout the map. The formulas for distortion are simpler when applied to regular cylindrical, conic (or conical), and polar azimuthal projections of the sphere. On each of these types of projections, scale is solely a function of latitude.

1.2.6 Calculations

The calculation of the distortions for the projections studied in the framework of this project is performed on WGS84, as this is the reference ellipsoid used in navigation based on Global Navigation Satellite Systems [GNSS]. Distortions on the WGS84 for the projections studied are computed utilizing the formulas provided by Snyder (1987) and Bugayevskiy and Snyder (1995) and double-checked in Matlab environment. It is pointed out that the difference in distortion values calculated on the ellipsoid and on the sphere is not significant when compared with the size of distortions. This implies that if - for any reason - there is a need for calculations on the sphere, this can be done without sacrificing the overall accuracy of the map/chart.

2   Study of map projection distortions for the Arctic Region

In this chapter, the distortions of Cylindrical, Conic and Azimuthal projections for the Arctic Region are analyzed and subsequently compared utilizing:

- Computation and portrayal of the Tissot indicatrix at different locations. They provide a generalized view of distortions from place to place [Maling, 1973]
- Computation and portrayal of isolines showing specific values of angular and area distortion. The pattern of isograms provides a two dimensional picture of how distortion varies from place to place, rather than the one-dimensional picture provided by a single graph [Maling, 1973]
- Computation and portrayal of graphs and tables displaying distortions and their values calculated at one degree interval of latitude. Graphs showing the particular scales plotted against latitude are utilized in assessing the relative merits of different map projections, which are suitable for a particular scope. The shape and gradient of each curve compared with others results to a comprehensive visual assessment of the projection characterized by the least distortion on a particular part of a map [Maling, 1973].

2.1   Cylindrical Projections

In this section, the use of cylindrical projections for the Arctic Region is examined. Four cylindrical projections are studied: Mercator (conformal), cylindrical equal area, equidistant cylindrical and Miller. A standard parallel at 65° is selected for all projections except for the Miller projection, which is calculated at the Equator. The study of projections and the evaluation of the distortions are based on:

- Maps portraying Tissot's ellipses of distortion: Figures 2 - 5
- Maps portraying isolines of angular and areal distortion: Figures 6 - 11
- Diagrams of Areal scale factor, Percentage of Areal distortion, Maximum angular distortion (ω), Scale factor along a meridian (h) and Scale factor along a parallel (k) calculated at one degree interval of latitude: Figures 12-16
- Table 2 showing values of Areal scale factor, Percentage of Areal distortion, Maximum angular distortion (ω), Scale factor along a meridian (h) and Scale factor along a parallel (k) calculated at one degree interval of latitude at one degree interval of latitude on WGS84

The following table shows the areas where the set limits for area and angular distortions are satisfied. In general areal and angular distortions for cylindrical projections are excessive and surpass the limits that make them visually unobservable except for certain parts of the study area where the set limits/tolerances are satisfied.

Projection	Limits for areal and angular distortion					
	Area: 8 - 10 %	Angle: 8° -10°	Area: 6 - 8 %	Angle: 6° - 8°	Area: 2%	Angle: 1°
Mercator	64° - 66°	conformal	64° - 65°	conformal	65°	conformal
Equidistant Cylindrical	62° - 67°	60° - 69°	63° - 66°	61° - 68°	65°	65°
Equal Area Cylindrical	equal area	63° - 67°	equal area	64° - 66°	equal area	65°
Miller	-	-	-	-	-	-

Table 1. Cylindrical projections: Criteria for angular and areal distortion are satisfied in certain parts of the study area ($\varphi_{min} - \varphi_{max}$).
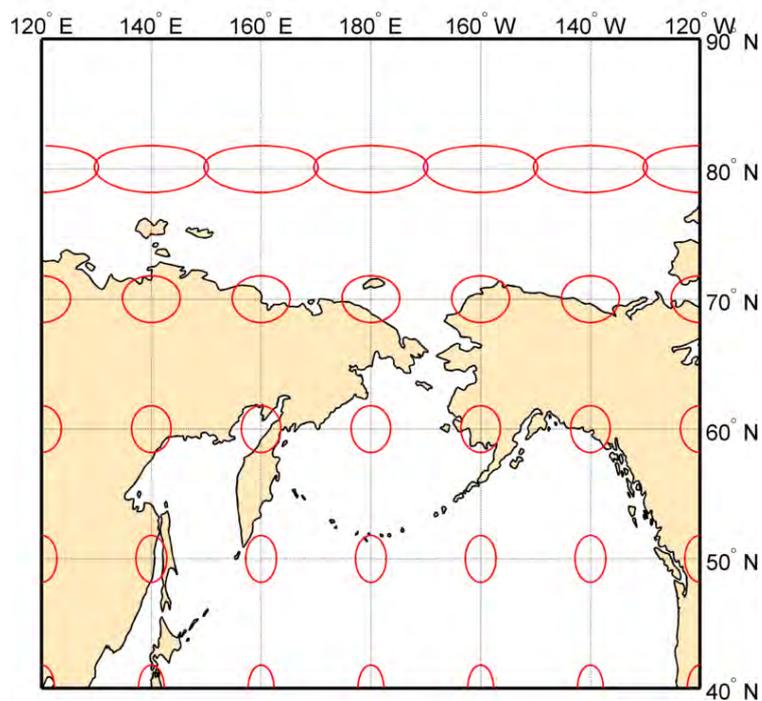
Figure 2. Equidistant Cylindrical projection – Tissot's Ellipses of distortion
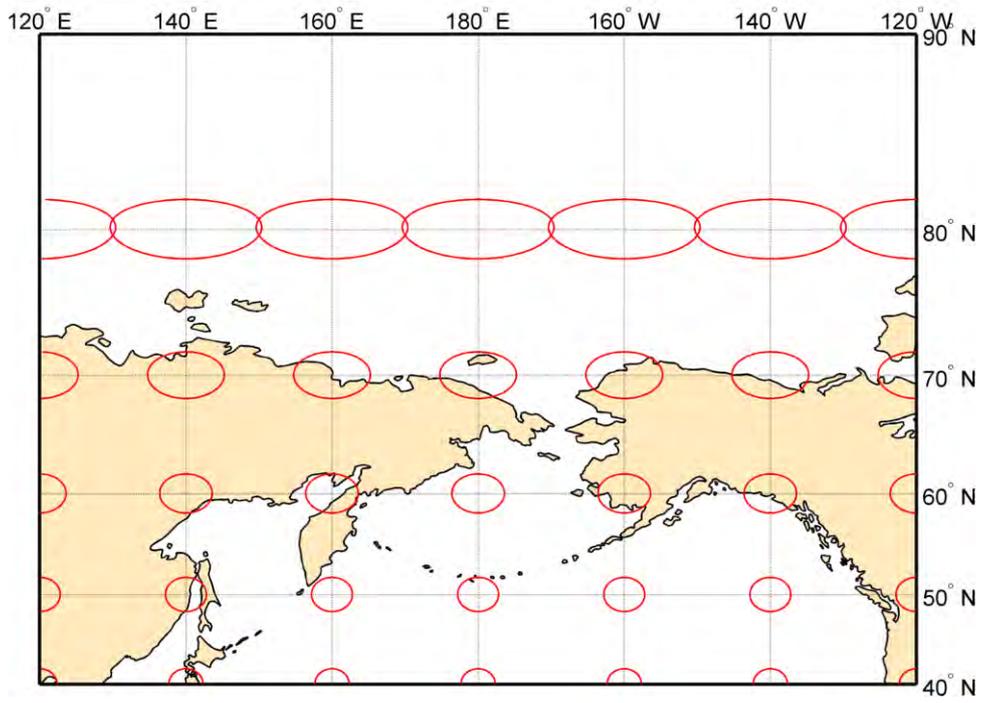
Figure 3. Miller projection – Tissot's Ellipses of distortion
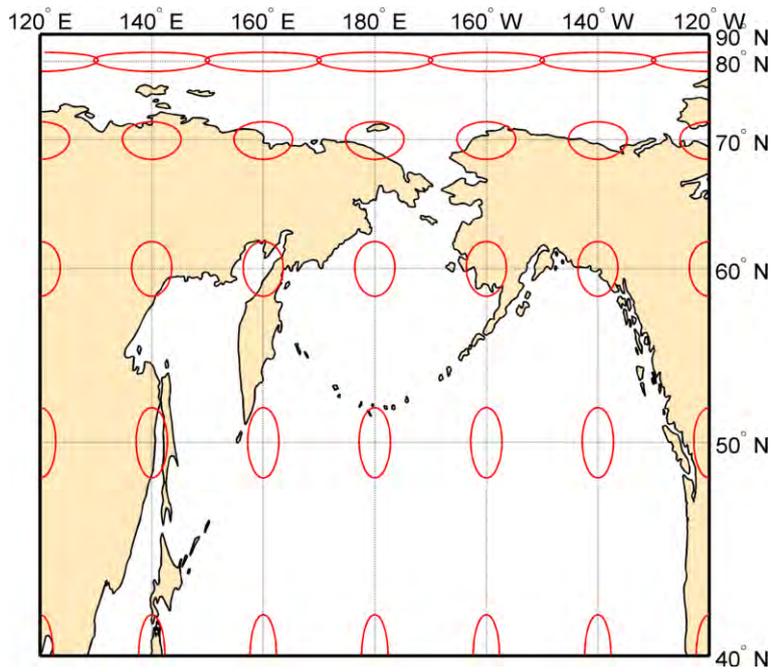
Figure 4. Equal Area Cylindrical projection - Tissot's Ellipses of distortion
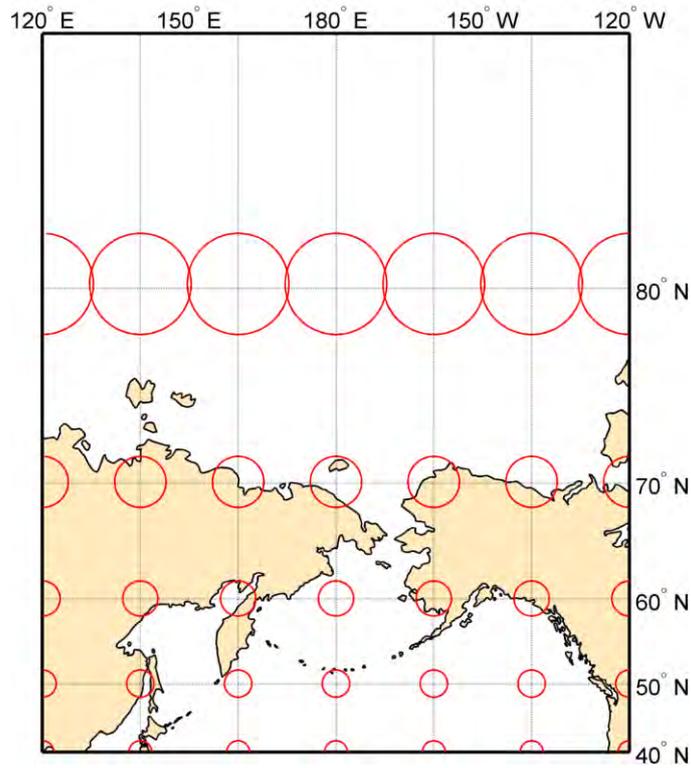
Figure 5. Mercator projection – Tissot's Ellipses of distortion
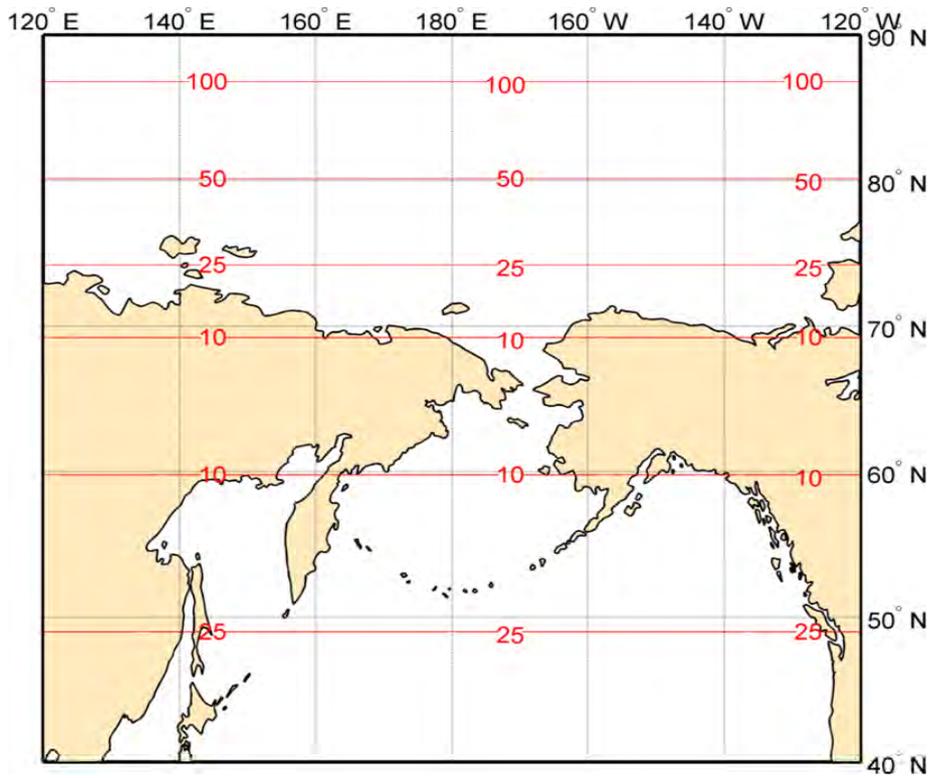
Figure 6. Cylindrical Equidistant projection – Isolines of maximum angular distortion
(10, 25, 50, 100 degrees)
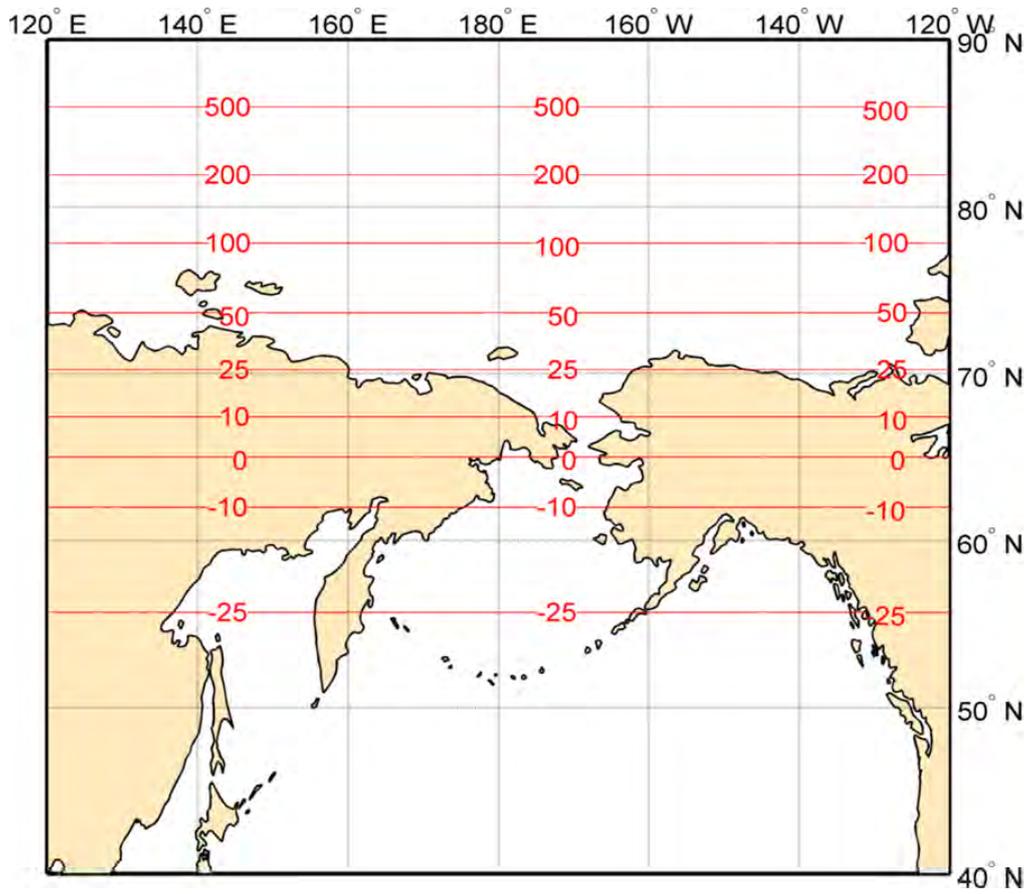
Figure 7. Cylindrical Equidistant projection – Isolines of areal distortion (%)
[-25, -10, 25, 50,100, 200, 500]
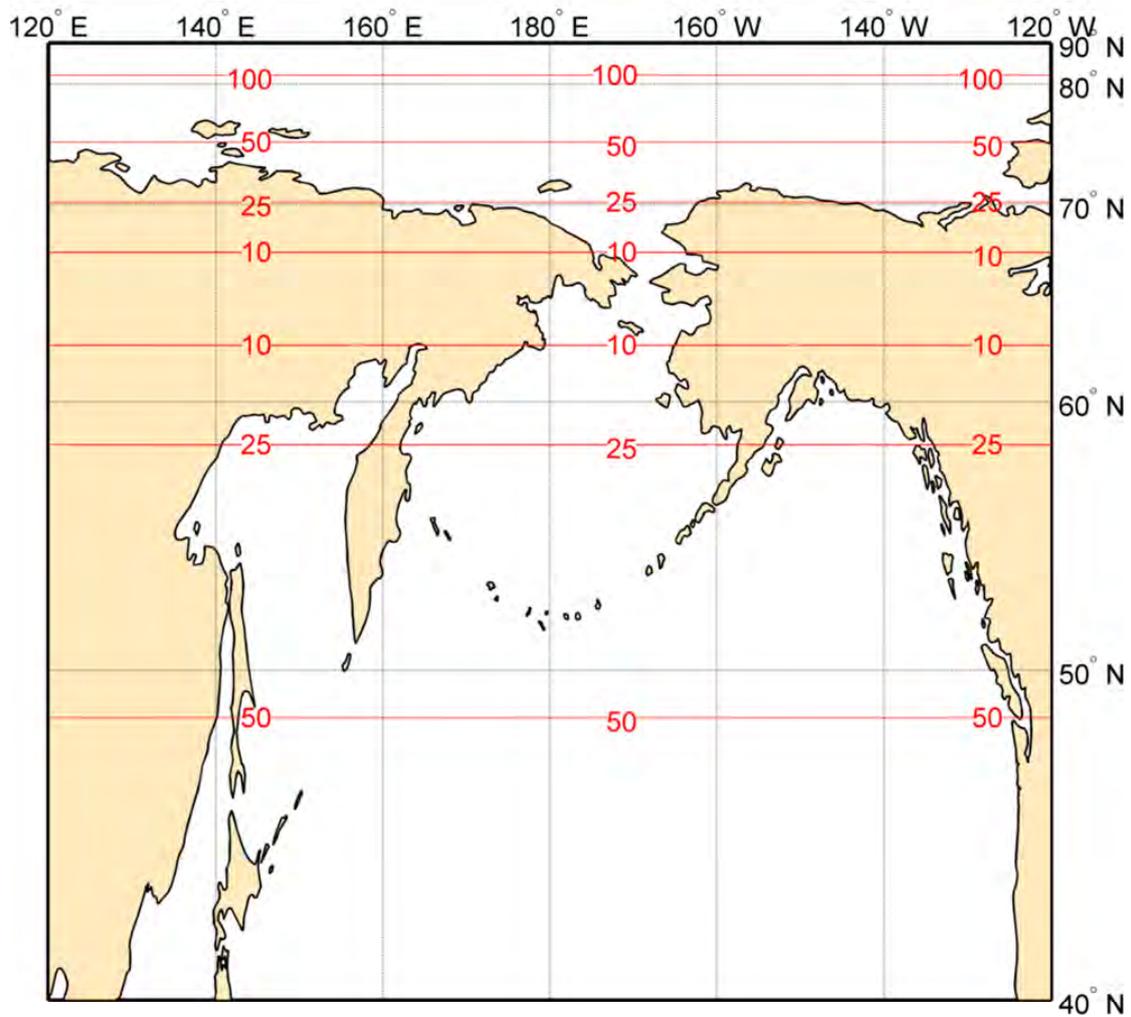
Figure 8. Equal Area Cylindrical projection – Isolines of maximum angular distortion
[10, 25, 50,100] (degrees)
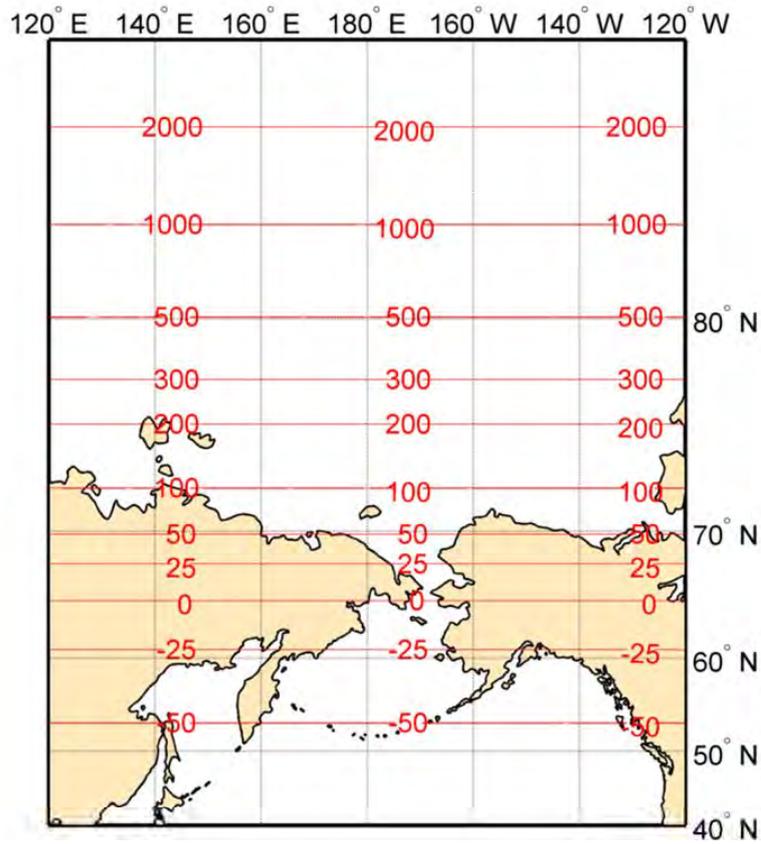
Figure 9. Mercator projection – Isolines of areal distortion (%)
[0, 25, 50, 100, 200, 300, 500, 1000, 2000]
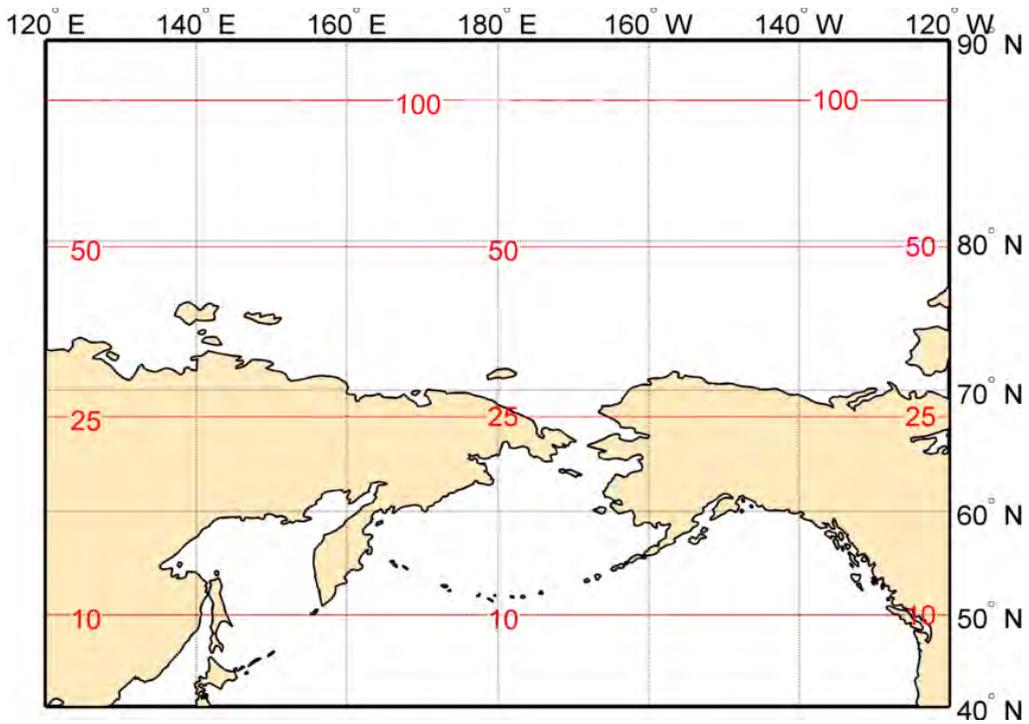
Figure 10. Miller projection - Isolines of maximum angular distortion [10, 25, 50, 100] (degrees)
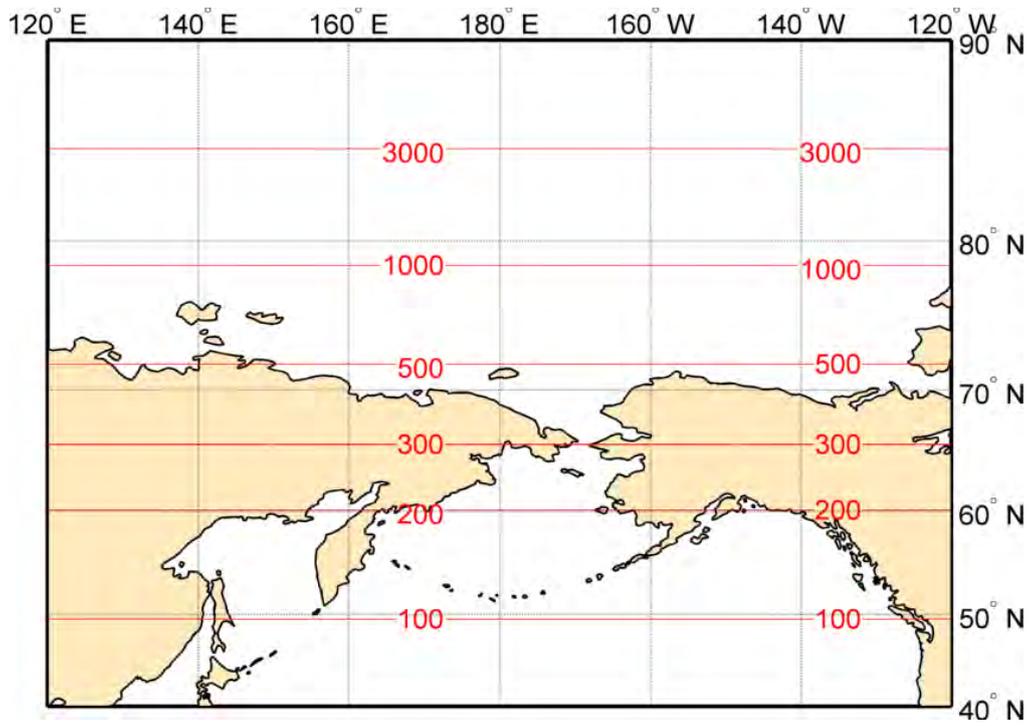
Figure 11. Miller projection – Isolines of areal distortion (%) [100, 200, 300, 500, 1000, 3000]
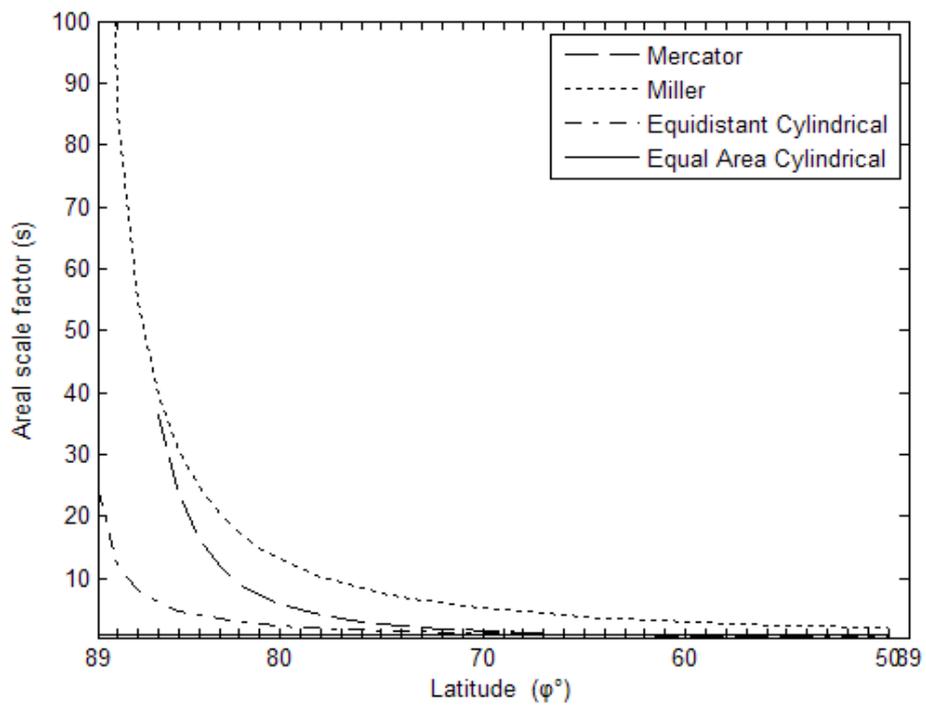
Figure 12. Cylindrical Projections: Area scale factor
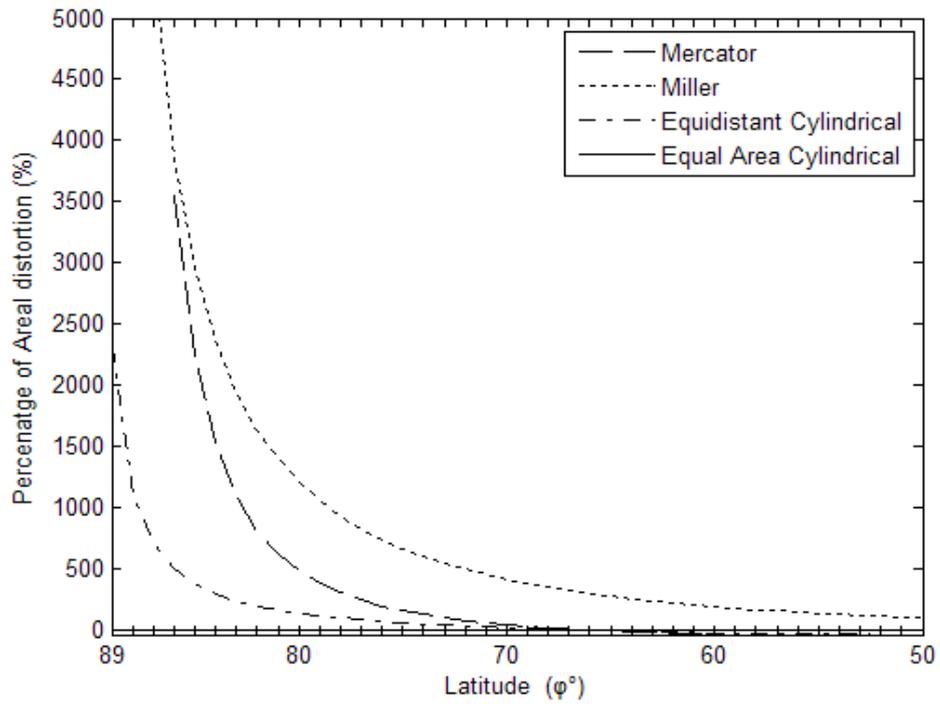
Figure 13. Cylindrical Projections: Percentage of Areal distortion
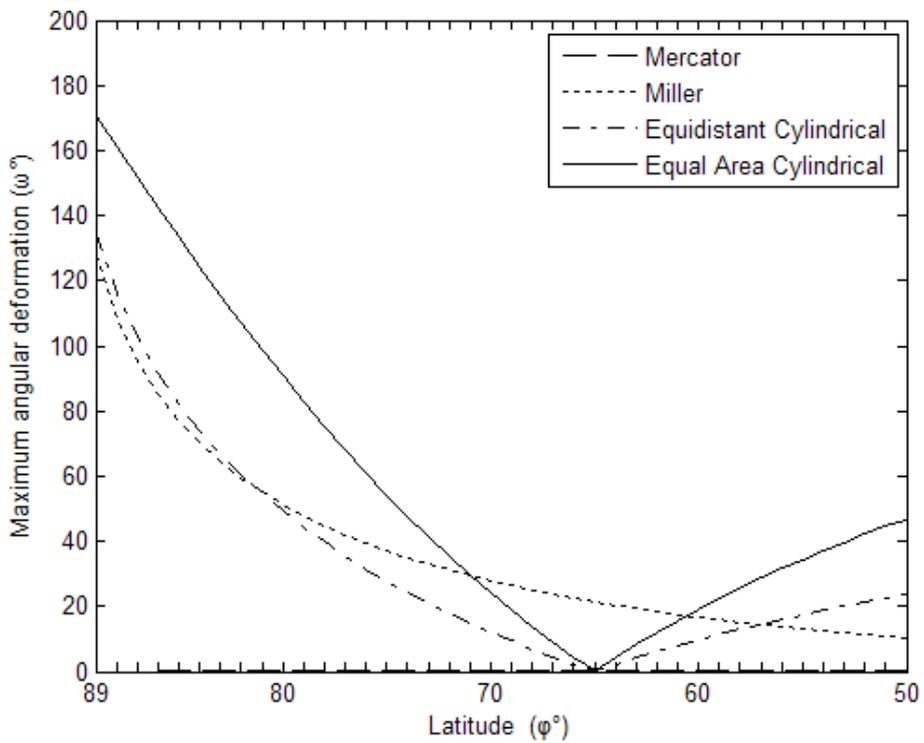
Figure 14. Cylindrical Projections: Maximum angular distortion (ω) in degrees
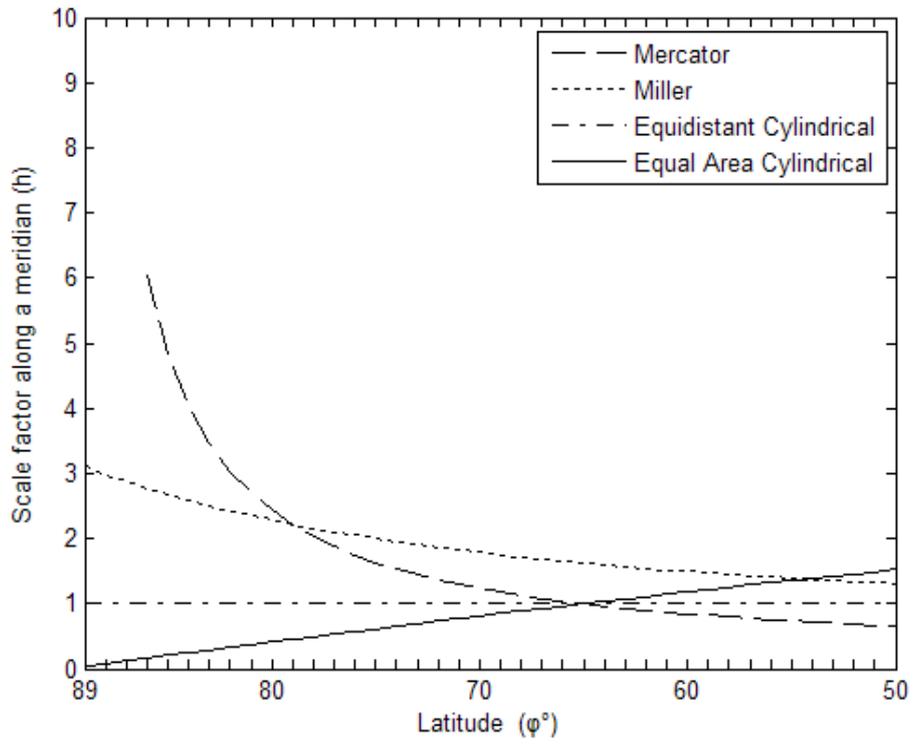
Figure 15. Cylindrical Projections: Scale factor along a meridian (h)
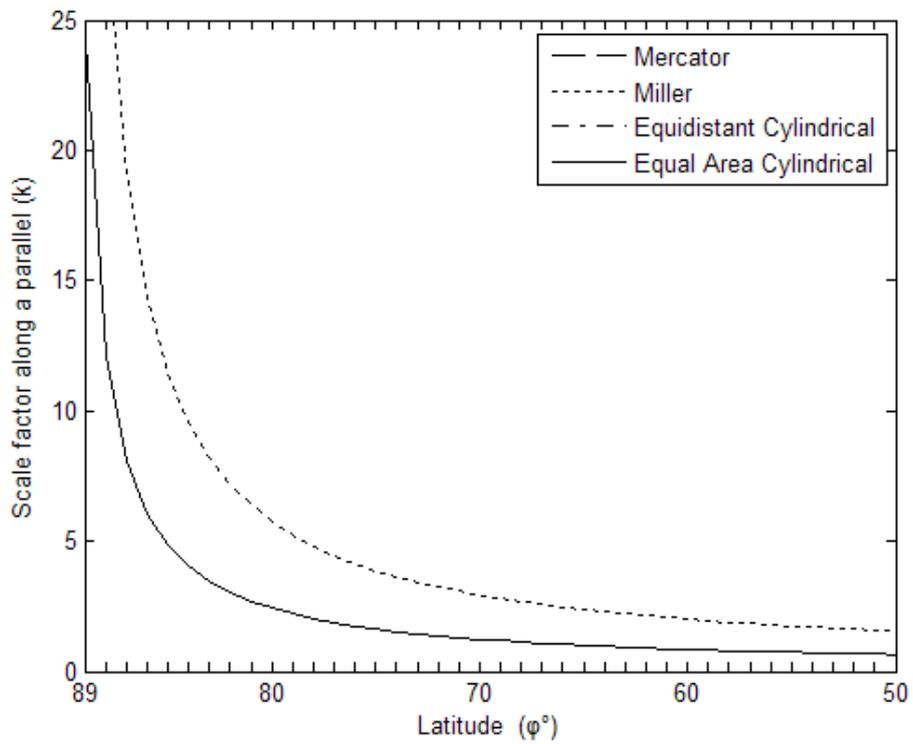
Figure 16. Cylindrical Projections: Scale factor along a parallel (h)

φ	mercator	(φo = 65)				equidistance (φo = 65)					equalarea (φo = 65)					miller				
	area	area %	angles	k	h	area	area %	angles	k	h	area	area %	angles	k	h	area	area %	angles	k	h
90																				
89						24,27	2327,49	134,03	24,23	1	1	0	170,54	24,20	0,04	176,61	17561,03	127,60	57,11	3,09
88						12,14	1113,93	115,84	12,12	1	1	0	161,11	12,10	0,08	84,85	8384,79	108,49	28,56	2,97
87						8,09	709,50	102,42	8,08	1	1	0	151,75	8,07	0,12	54,45	5345,10	95,28	19,04	2,86
86	36,68	3568,08	0	6,06	6,06	6,07	507,34	91,52	6,06	1	1	0	142,49	6,05	0,17	39,38	3837,93	85,15	14,29	2,76
85	23,62	2261,50	0	4,86	4,86	4,86	386,10	82,27	4,85	1	1	0	133,36	4,85	0,21	30,43	2942,67	77,00	11,44	2,66
84	16,42	1541,80	0	4,05	4,05	4,05	305,31	74,19	4,05	1	1	0	124,40	4,04	0,25	24,53	2352,57	70,21	9,54	2,57
83	12,08	1107,85	0	3,48	3,48	3,48	247,65	67,02	3,47	1	1	0	115,62	3,47	0,29	20,36	1936,27	64,45	8,18	2,49
82	9,26	826,20	0	3,04	3,04	3,04	204,43	60,56	3,04	1	1	0	107,05	3,04	0,33	17,28	1628,15	59,47	7,16	2,41
81	7,33	633,10	0	2,71	2,71	2,71	170,84	54,69	2,70	1	1	0	98,71	2,70	0,37	14,92	1391,79	55,10	6,37	2,34
80	5,95	494,99	0	2,44	2,44	2,44	144,00	49,32	2,44	1	1	0	90,61	2,43	0,41	13,05	1205,41	51,25	5,74	2,27
79	4,93	392,80	0	2,22	2,22	2,22	122,06	44,36	2,22	1	1	0	82,76	2,21	0,45	11,55	1055,15	47,81	5,22	2,21
78	4,15	315,08	0	2,04	2,04	2,04	103,80	39,78	2,03	1	1	0	75,18	2,03	0,49	10,32	931,81	44,71	4,79	2,15
77	3,55	254,60	0	1,88	1,88	1,88	88,37	35,51	1,88	1	1	0	67,86	1,88	0,53	9,29	829,03	41,91	4,43	2,10
76	3,07	206,61	0	1,75	1,75	1,75	75,16	31,52	1,75	1	1	0	60,81	1,75	0,57	8,42	742,29	39,37	4,12	2,04
75	2,68	167,90	0	1,64	1,64	1,64	63,73	27,78	1,63	1	1	0	54,02	1,63	0,61	7,68	668,27	37,04	3,85	1,99
74	2,36	136,22	0	1,54	1,54	1,54	53,74	24,27	1,53	1	1	0	47,51	1,53	0,65	7,05	604,52	34,90	3,62	1,95
73	2,10	109,97	0	1,45	1,45	1,45	44,95	20,96	1,45	1	1	0	41,26	1,45	0,69	6,49	549,14	32,93	3,41	1,90
72	1,88	87,97	0	1,37	1,37	1,37	37,14	17,83	1,37	1	1	0	35,27	1,37	0,73	6,01	500,68	31,11	3,23	1,86
71	1,69	69,35	0	1,30	1,30	1,30	30,18	14,87	1,30	1	1	0	29,53	1,30	0,77	5,58	458,01	29,42	3,06	1,82
70	1,53	53,47	0	1,24	1,24	1,24	23,92	12,07	1,24	1	1	0	24,04	1,24	0,81	5,20	420,20	27,84	2,92	1,78
69	1,40	39,79	0	1,18	1,18	1,18	18,27	9,41	1,18	1	1	0	18,79	1,18	0,85	4,87	386,52	26,37	2,78	1,75
68	1,28	27,95	0	1,13	1,13	1,13	13,15	6,88	1,13	1	1	0	13,77	1,13	0,89	4,56	356,38	25,00	2,66	1,71
67	1,18	17,62	0	1,08	1,08	1,08	8,49	4,47	1,08	1	1	0	8,97	1,08	0,92	4,29	329,29	23,71	2,55	1,68
66	1,09	8,55	0	1,04	1,04	1,04	4,22	2,17	1,04	1	1	0	4,38	1,04	0,96	4,05	304,84	22,50	2,45	1,65
65	1	0,55	0	1	1	1	0,31	0	1	1	1	0	0	1	1	3,83	282,69	21,36	2,36	1,62
64	0,93	-6,53	0	0,97	0,97	0,97	-3,29	2,11	0,97	1	1	0	4,19	0,96	1,04	3,63	262,56	20,29	2,27	1,59
63	0,87	-12,85	0	0,93	0,93	0,93	-6,61	4,12	0,93	1	1	0	8,19	0,93	1,07	3,44	244,21	19,27	2,20	1,57
62	0,82	-18,49	0	0,90	0,90	0,90	-9,69	6,03	0,90	1	1	0	12,01	0,90	1,11	3,27	227,43	18,31	2,12	1,54
61	0,76	-23,56	0	0,87	0,87	0,87	-12,54	7,87	0,87	1	1	0	15,66	0,87	1,15	3,12	212,04	17,40	2,06	1,52
60	0,72	-28,13	0	0,85	0,85	0,85	-15,20	9,63	0,85	1	1	0	19,15	0,85	1,18	2,98	197,90	16,54	1,99	1,49
59	0,68	-32,26	0	0,82	0,82	0,82	-17,67	11,31	0,82	1	1	0	22,48	0,82	1,22	2,85	184,87	15,72	1,94	1,47
58	0,64	-36,00	0	0,80	0,80	0,80	-19,98	12,93	0,80	1	1	0	25,67	0,80	1,25	2,73	172,84	14,94	1,88	1,45
57	0,61	-39,41	0	0,78	0,78	0,78	-22,14	14,49	0,78	1	1	0	28,72	0,78	1,29	2,62	161,71	14,20	1,83	1,43
56	0,57	-42,52	0	0,76	0,76	0,76	-24,16	15,98	0,76	1	1	0	31,63	0,76	1,32	2,51	151,39	13,49	1,78	1,41
55	0,55	-45,36	0	0,74	0,74	0,74	-26,06	17,42	0,74	1	1	0	34,41	0,74	1,36	2,42	141,81	12,82	1,74	1,39
54	0,52	-47,96	0	0,72	0,72	0,72	-27,84	18,80	0,72	1	1	0	37,07	0,72	1,39	2,33	132,90	12,18	1,70	1,37
53	0,50	-50,35	0	0,70	0,70	0,70	-29,52	20,13	0,70	1	1	0	39,62	0,70	1,42	2,25	124,60	11,57	1,66	1,35
52	0,47	-52,56	0	0,69	0,69	0,69	-31,10	21,41	0,69	1	1	0	42,06	0,69	1,46	2,17	116,85	10,98	1,62	1,34
51	0,45	-54,59	0	0,67	0,67	0,67	-32,59	22,64	0,67	1	1	0	44,39	0,67	1,49	2,10	109,61	10,42	1,59	1,32
50	0,44	-56,47	0	0,66	0,66	0,66	-34,00	23,82	0,66	1	1	0	46,62	0,66	1,52	2,03	102,84	9,88	1,55	1,31

Table 2. Cylindrical projections: Values for Area Scale Error, Percentage of areal distortion, Maximum angular deformation (ω) in decimal degrees, Scale factor along a parallel (k), Scale factor along a meridian (h) for Mercator, Equidistant Cylindrical, Equal Area Cylindrical and Miller projection.

2.2   Conic Projections

In this section, the use of conic projections for the portrayal of the Arctic Region is examined. Three conic projections are studied: Lambert Conformal, Conic Equal Area, and Conic Equidistant. In order to minimize the distortion two standard parallels are selected. The latitudes φ_1 and φ_2 of the standard parallels are selected by using formulas [1] and [2] introduced by Kavraisky and referred by Bugayevskiy and Snyder (1995) and Maling (1973).

$$\varphi_1 = \varphi_S + \Delta\varphi \quad \text{[1]}$$

$$\varphi_2 = \varphi_N - \Delta\varphi \quad \text{[2]}$$

where:

$$\Delta\varphi = \frac{\varphi_N - \varphi_S}{k}$$

φ_S and φ_N are the bounding parallels of the map

The use of the constant K leads to the choice of the suitable standard parallels for conical projections. This constant may vary according to the shape of the area to be mapped as follows:

a. Small extend in latitude but large extend in longitude: K = 7
b. Rectangular outline with longer axis north-south: K = 5
c. Circular or elliptical outline: K = 4
d. Square outline: K = 3

By selecting K = 5 two standard parallels at 58° and 82° are applied to the projections studied. The following table shows the areas where the set limits for area and angular distortions are satisfied. In general areal and angular distortions for conic projections are lower than those of the cylindrical projections but still exceed the limits that make them visually unobservable except for certain sub areas where the set limits/tolerances are satisfied.

The study of projections and the evaluation of the distortions are based on:

- Maps portraying Tissot's ellipses of distortion: Figures 17 - 19
- Maps portraying isolines of angular and areal distortion: Figures 20 - 23
- Diagrams of Areal scale factor, Percentage of Areal distortion, Maximum angular distortion (ω), Scale factor along a meridian (h) and Scale factor along a parallel (k) calculated at one degree interval of latitude: Figures 24 - 28
- Table 2 showing values of Areal scale factor, Percentage of Areal distortion, Maximum angular distortion (ω), Scale factor along a meridian (h) and Scale factor along a parallel (k) calculated at one degree interval of latitude at one degree interval of latitude on WGS84.

Projection	Limits for angular and areal distortions					
	Area: 8 -10%	Angle: 8° -10°	Area: 6 - 8%	Angle: 6° - 8°	Area: 2%	Angle: 1°
Lambert	50°- 87°	conformal	50° - 86°	conformal	80° - 83° & 56° - 61°	conformal
Conic Equal Area	equal area	50° - 85°	equal area	50° - 85°	equal area	55° - 62° & 81° - 82°
Conic Equidistant	50° - 86°	50° - 88°	50° - 86°	50° - 87°	52° - 67° & 77° - 84°	53° - 65° & 79° - 83°

Table 3. Conic Projections: Criteria for angular and areal distortion are satisfied in certain parts of the study area (φ_{min} – φ_{max}).

Figure 17. Equal Area Conic – Tissot's Ellipses of distortion
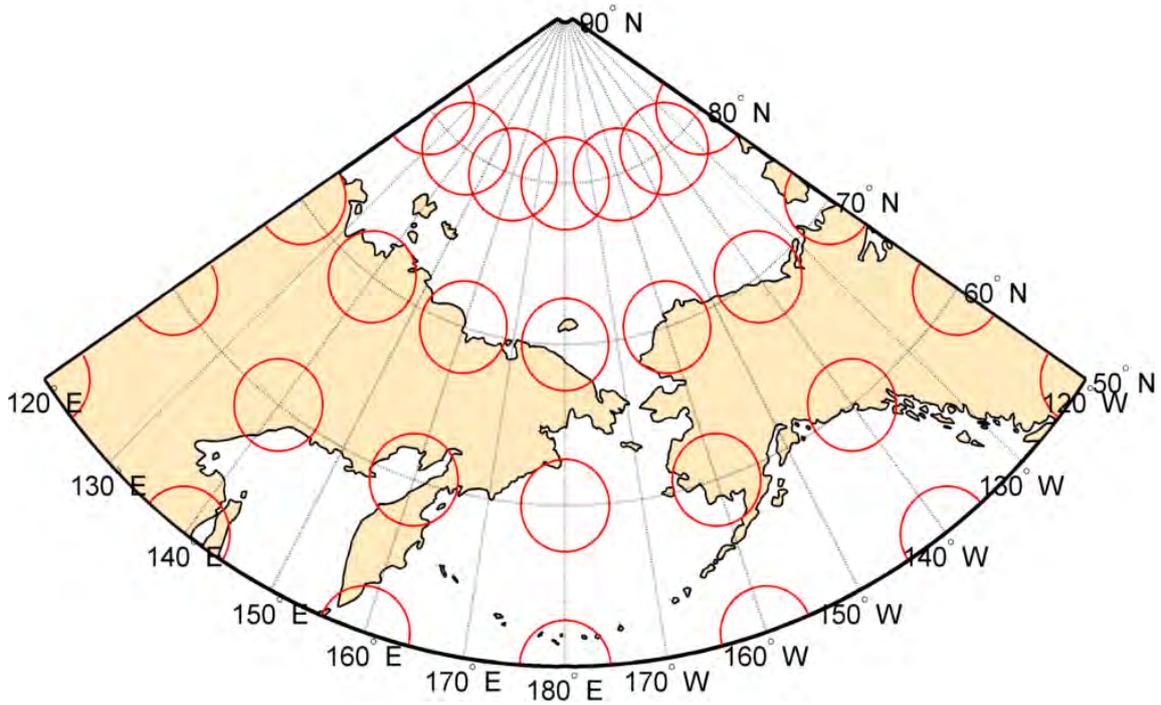
Figure 18. Equidistant Conic – Tissot's Ellipses of distortion
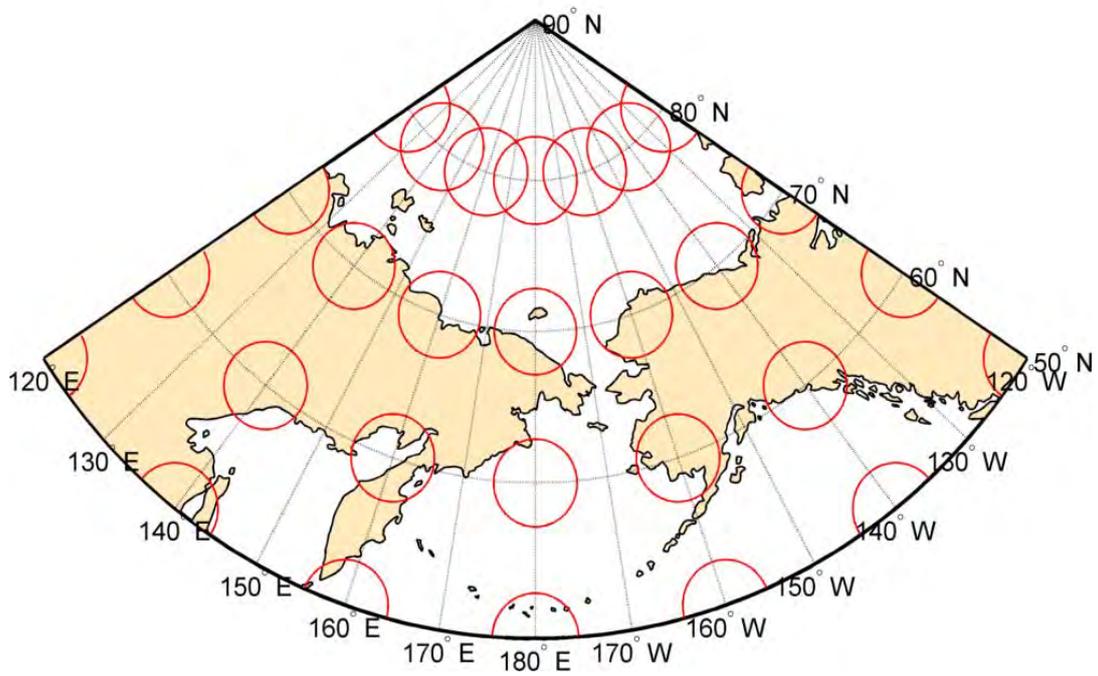
Figure 19. Lambert (conformal) Conic – Tissot's Ellipses of distortion
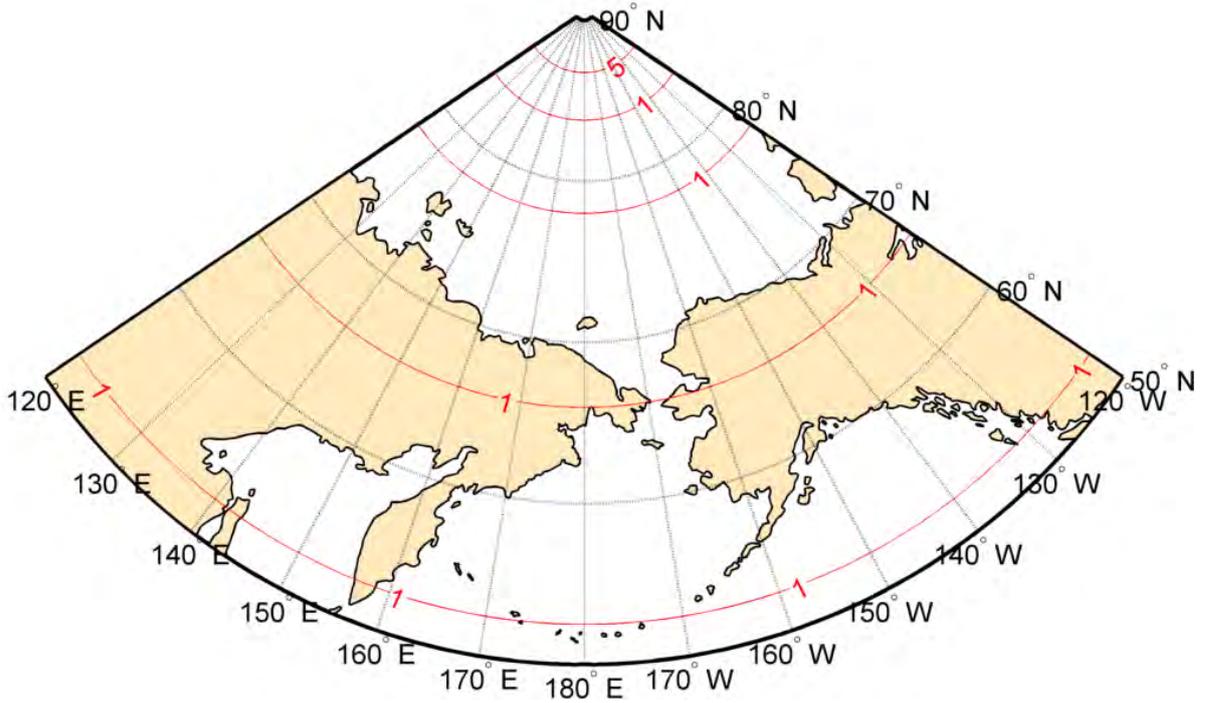
Figure 20. Conic Equidistant projection – Isolines of maximum angular distortion (degrees) [1, 5]
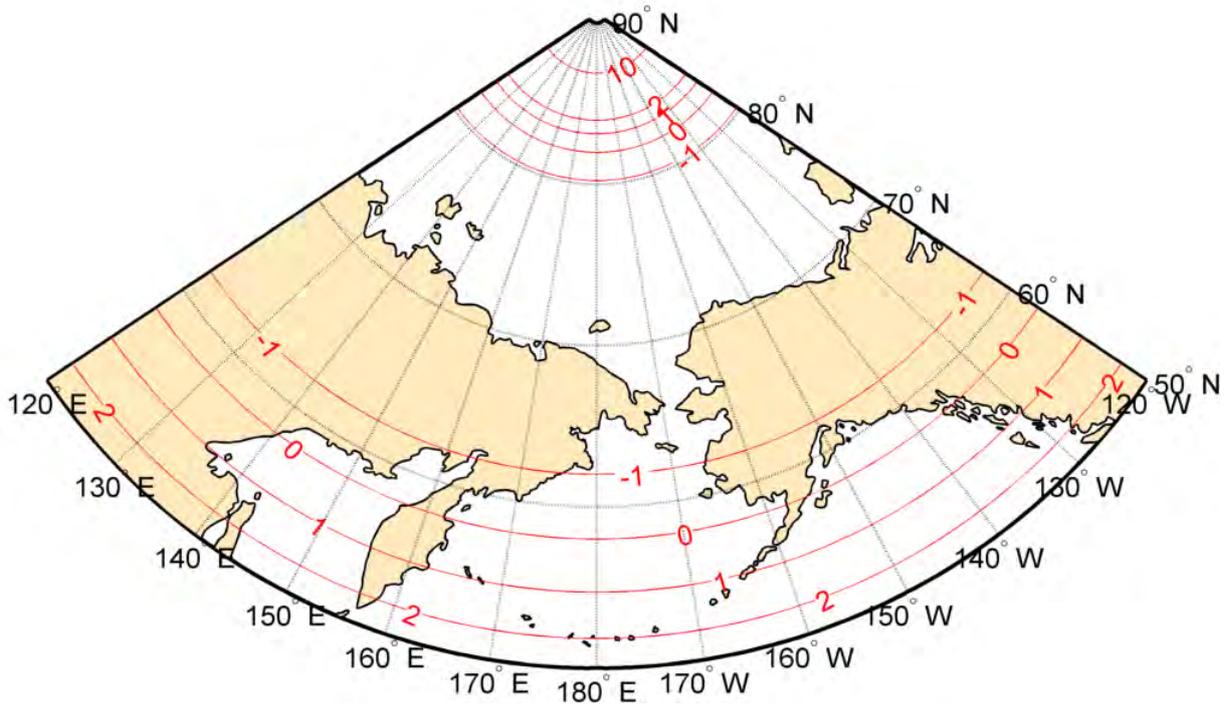
Figure 21. Conic Equidistant projection – Isolines of areal distortion (%) [0, 1, 2, 10]
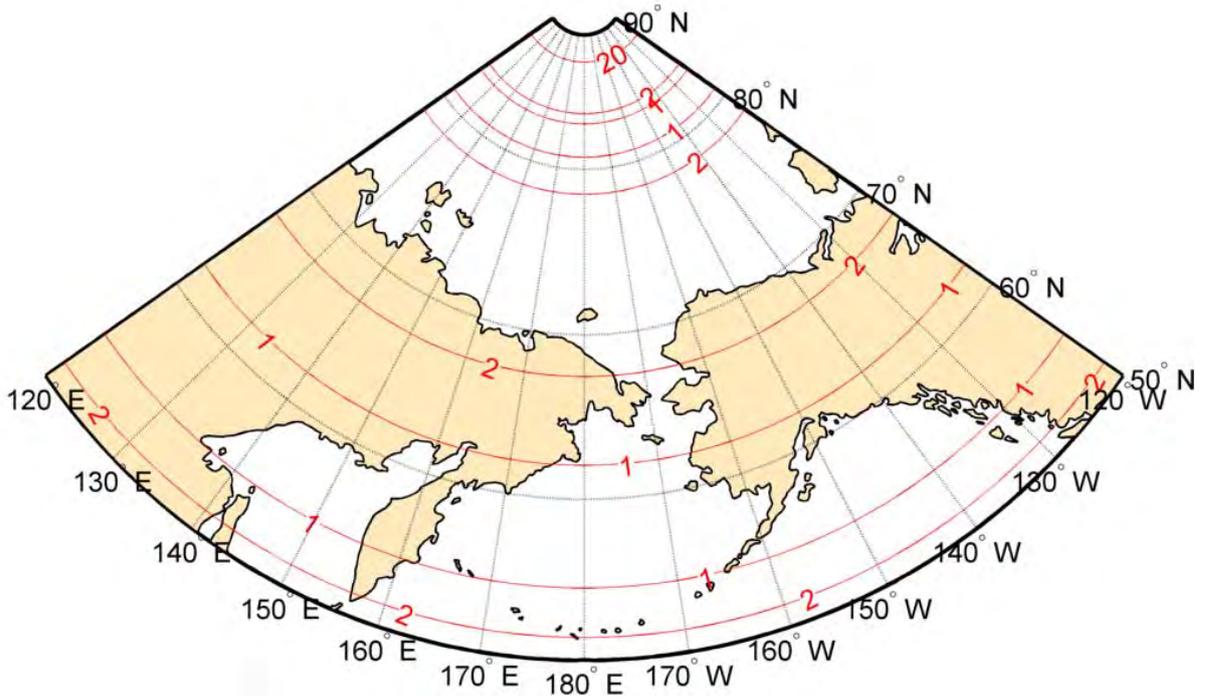
Figure 22. Conic Equal Area projection – Isolines of maximum angular distortions [1, 2, 20] (degrees)
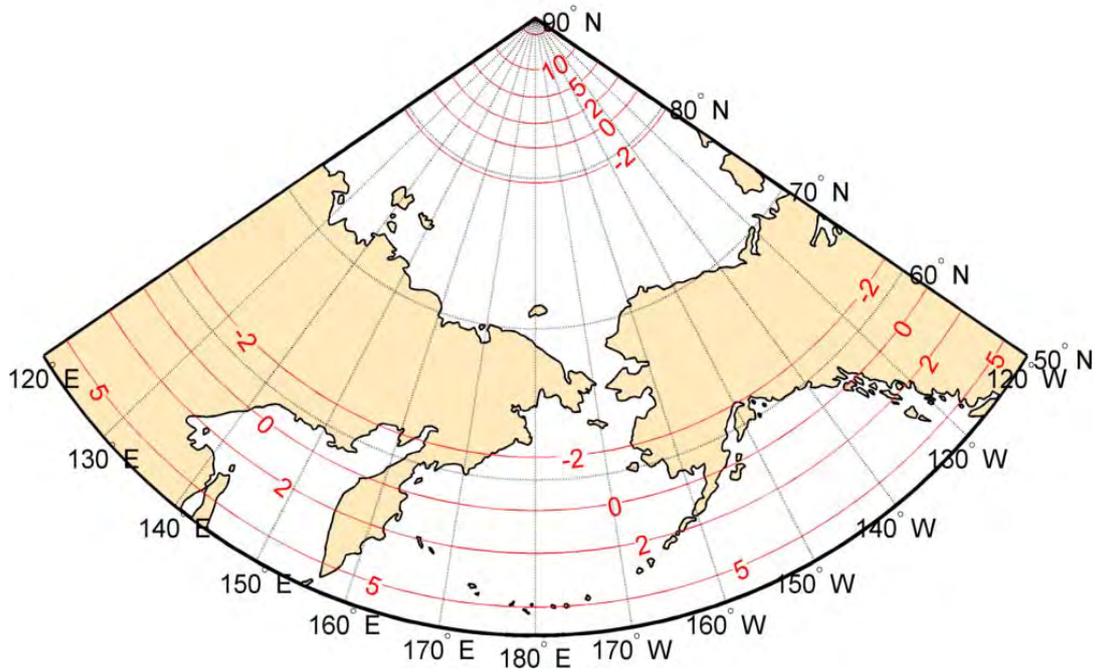
Figure 23. Lambert Conic projection – Isolines of areal distortion (%) [0, 2, 5, 10]
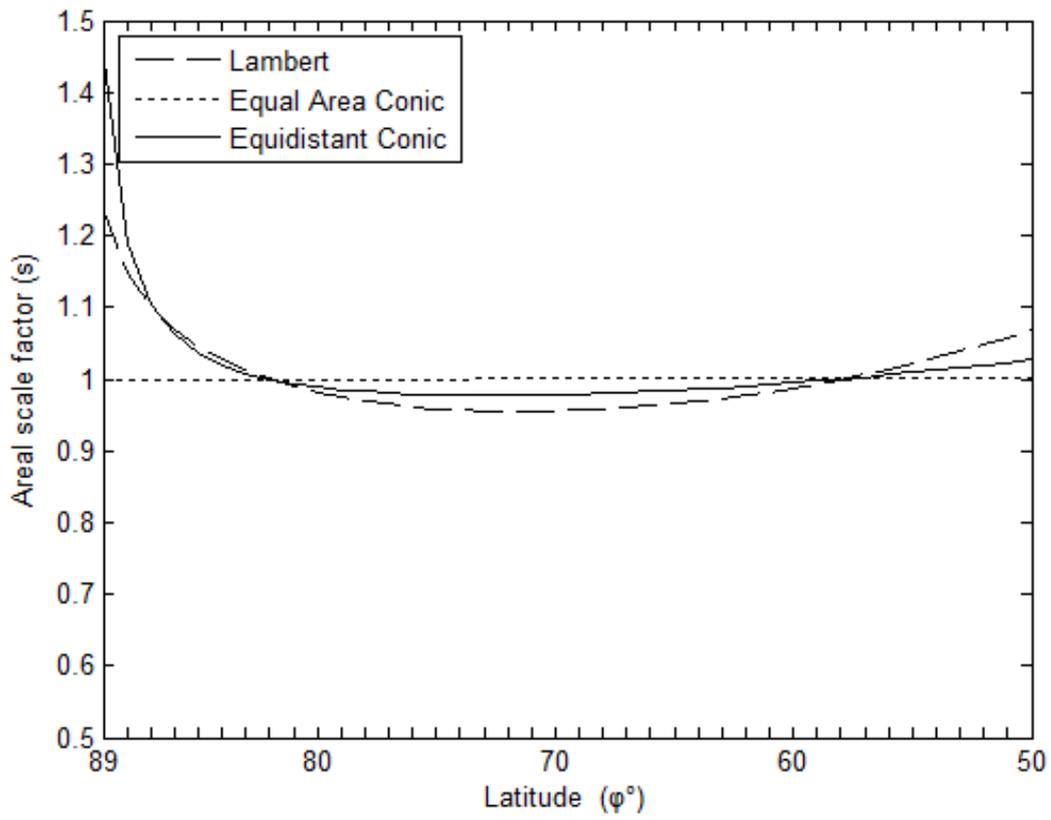
Figure 24. Conic projections: Areal scale factor
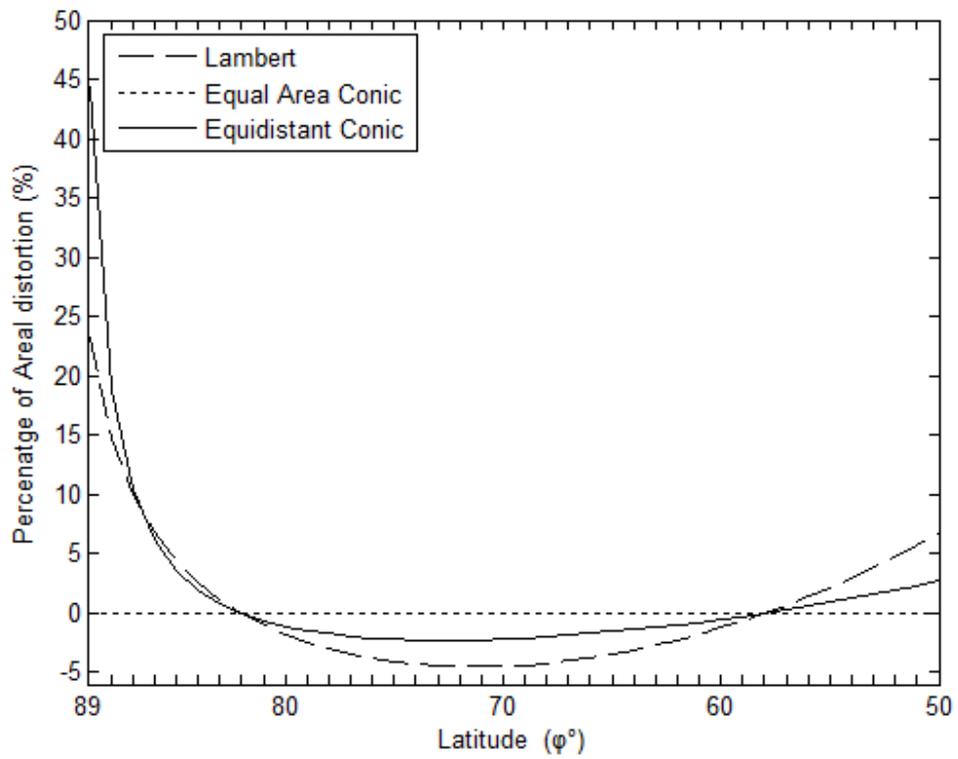
Figure 25. Conic projections: Percentage of areal distortion
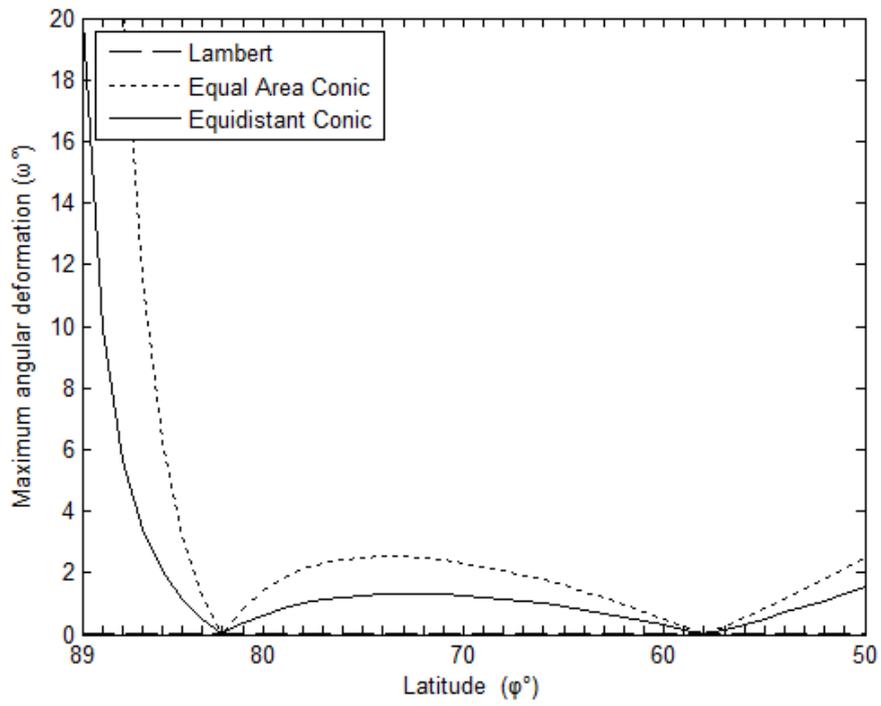
Figure 26. Conic projections: Maximum angular distortion (ω) in degrees.
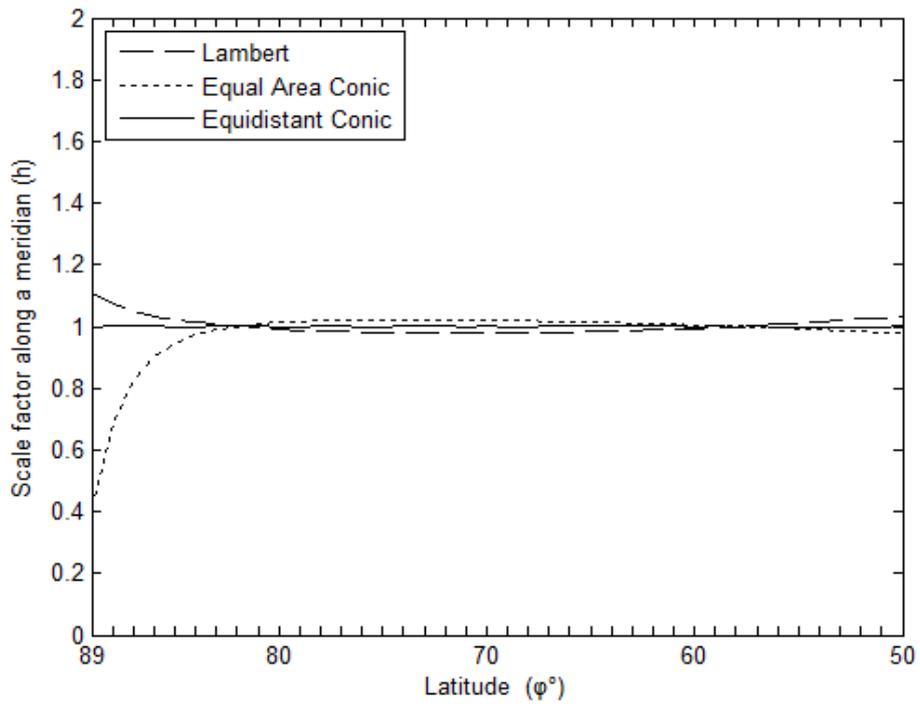
Figure 27. Conic projections: Scale factor along a meridian (h)
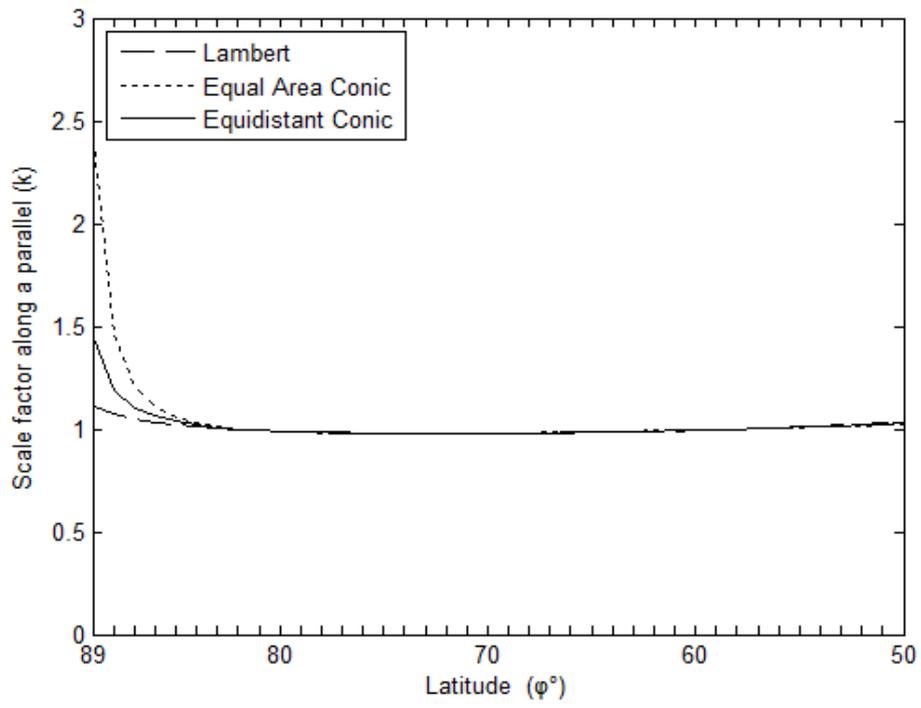
Figure 28. Conic projections: Scale factor along a parallel (k)

φ	lambert					equalarea					eqdiz				
	area	area %	angles	k	h	area	area %	angles	k	h	area	area %	angles	k	h
90						1	0								
89	1,23	23,24	0	1,11	1,11	1	0	89,60	2,40	0,42	1,44	44,40	20,93	1,44	1
88	1,15	14,64	0	1,07	1,07	1	0	42,40	1,46	0,68	1,19	18,86	9,89	1,19	1
87	1,10	9,94	0	1,05	1,05	1	0	21,53	1,21	0,83	1,10	10,37	5,65	1,10	1
86	1,07	6,78	0	1,03	1,03	1	0	11,56	1,11	0,90	1,06	6,15	3,42	1,06	1
85	1,04	4,45	0	1,02	1,02	1	0	6,26	1,06	0,95	1,04	3,64	2,05	1,04	1
84	1,03	2,64	0	1,01	1,01	1	0	3,18	1,03	0,97	1,02	1,99	1,13	1,02	1
83	1,01	1,18	0	1,01	1,01	1	0	1,26	1,01	0,99	1,01	0,84	0,48	1,01	1
82	1	0	0	1	1	1	0	0	1	1	1	0	0	1	1
81	0,99	-0,98	0	1,00	1,00	1	0	0,85	0,99	1,01	0,99	-0,63	0,36	0,99	1
80	0,98	-1,79	0	0,99	0,99	1	0	1,44	0,99	1,01	0,99	-1,10	0,63	0,99	1
79	0,98	-2,46	0	0,99	0,99	1	0	1,84	0,98	1,02	0,99	-1,46	0,84	0,99	1
78	0,97	-3,01	0	0,98	0,98	1	0	2,13	0,98	1,02	0,98	-1,74	1,00	0,98	1
77	0,97	-3,46	0	0,98	0,98	1	0	2,32	0,98	1,02	0,98	-1,94	1,12	0,98	1
76	0,96	-3,82	0	0,98	0,98	1	0	2,44	0,98	1,02	0,98	-2,09	1,21	0,98	1
75	0,96	-4,10	0	0,98	0,98	1	0	2,51	0,98	1,02	0,98	-2,19	1,27	0,98	1
74	0,96	-4,30	0	0,98	0,98	1	0	2,53	0,98	1,02	0,98	-2,25	1,31	0,98	1
73	0,96	-4,43	0	0,98	0,98	1	0	2,52	0,98	1,02	0,98	-2,28	1,32	0,98	1
72	0,95	-4,50	0	0,98	0,98	1	0	2,48	0,98	1,02	0,98	-2,27	1,32	0,98	1
71	0,95	-4,51	0	0,98	0,98	1	0	2,41	0,98	1,02	0,98	-2,24	1,30	0,98	1
70	0,96	-4,46	0	0,98	0,98	1	0	2,32	0,98	1,02	0,98	-2,18	1,27	0,98	1
69	0,96	-4,36	0	0,98	0,98	1	0	2,20	0,98	1,02	0,98	-2,10	1,22	0,98	1
68	0,96	-4,20	0	0,98	0,98	1	0	2,07	0,98	1,02	0,98	-2,00	1,16	0,98	1
67	0,96	-4,00	0	0,98	0,98	1	0	1,93	0,98	1,02	0,98	-1,87	1,09	0,98	1
66	0,96	-3,74	0	0,98	0,98	1	0	1,77	0,98	1,02	0,98	-1,73	1,00	0,98	1
65	0,97	-3,44	0	0,98	0,98	1	0	1,59	0,99	1,01	0,98	-1,57	0,91	0,98	1
64	0,97	-3,08	0	0,98	0,98	1	0	1,40	0,99	1,01	0,99	-1,40	0,81	0,99	1
63	0,97	-2,69	0	0,99	0,99	1	0	1,20	0,99	1,01	0,99	-1,20	0,70	0,99	1
62	0,98	-2,24	0	0,99	0,99	1	0	0,98	0,99	1,01	0,99	-0,99	0,57	0,99	1
61	0,98	-1,75	0	0,99	0,99	1	0	0,75	0,99	1,01	0,99	-0,77	0,44	0,99	1
60	0,99	-1,21	0	0,99	0,99	1	0	0,51	1,00	1,00	0,99	-0,53	0,30	0,99	1
59	0,99	-0,63	0	1,00	1,00	1	0	0,26	1,00	1,00	1,00	-0,27	0,16	1,00	1
58	1	0	0	1	1	1	0	0	1	1	1	0	0	1	1
57	1,01	0,68	0	1,00	1,00	1	0	0,27	1,00	1,00	1,00	0,29	0,16	1,00	1
56	1,01	1,40	0	1,01	1,01	1	0	0,56	1,00	1,00	1,01	0,59	0,34	1,01	1
55	1,02	2,17	0	1,01	1,01	1	0	0,85	1,01	0,99	1,01	0,90	0,52	1,01	1
54	1,03	2,99	0	1,01	1,01	1	0	1,16	1,01	0,99	1,01	1,23	0,70	1,01	1
53	1,04	3,86	0	1,02	1,02	1	0	1,48	1,01	0,99	1,02	1,57	0,90	1,02	1
52	1,05	4,78	0	1,02	1,02	1	0	1,81	1,02	0,98	1,02	1,93	1,10	1,02	1
51	1,06	5,75	0	1,03	1,03	1	0	2,14	1,02	0,98	1,02	2,31	1,31	1,02	1
50	1,07	6,77	0	1,03	1,03	1	0	2,49	1,02	0,98	1,03	2,69	1,53	1,03	1

Table 4. Conic projections: Values for areal distortion, Percentage of areal distortion, Maximum angular distortion (ω) in degrees, Scale factor along a parallel (k) and Scale factor along a meridian (k) for Lambert Conic, Conic Equal Area and Conic Equidistant projections.

2.3 Azimuthal projections

In this section, the use of Azimuthal family of projections for the portrayal of the Arctic Region is examined. The traditional approach to the choice of a projection class in most of the elementary text books suggests that "if a map is required to show one of the Polar Regions, an azimuthal projection should be used" [Pearson 1990]. In the "Table of Suitable projections" appearing in Steers and Debenham (1970) the Azimuthal Equal Area and the Azimuthal Equidistant are proposed for the Polar Regions. Furthermore it is stated that: "A great many projections e.g. gnomonic, stereographic are fairly suitable, but the two mentioned are probably best". According to Maling (1973), an equidistant map/chart is a useful compromise between the two extremes represented by conformal and equal-area maps/charts. The areal distortion of an equidistant map/chart increases more slowly than that of a conformal map/chart. The maximum angular deformation of an equidistant map/chart increases more slowly than that of an equal area map/chart. Consequently equidistant map projections are used often in atlas maps, strategic planning maps and similar representations of large parts of the earth's surface in which it is not necessary to preserve either of the two other properties. Especially for the azimuthal equidistant projection, it is stated that there is also a close relationship between the distortion characteristics of the equidistant and the Airy projection, which is the minimum error projection of this class (Mailing 1973).

Five Azimuthal projections are studied: Orthographic, Stereographic (conformal), Azimuthal Equal Area, Azimuthal Equidistant and Gnomonic. All projections are applied with the polar aspect. The study of projections and the evaluation of the distortions are based on:

- Maps portraying Tissot's ellipses of distortion: Figures 29 - 32
- Maps portraying isolines of angular and areal distortion: Figures 33 - 40
- Diagrams of Areal scale factor, Percentage of Areal distortion, Maximum angular distortion (ω), Scale factor along a meridian (h) and Scale factor along a parallel (k) calculated at one degree interval of latitude: Figures 41 - 45
- Table 6 showing values of Areal scale factor, Percentage of Areal distortion, Maximum angular distortion (ω), Scale factor along a meridian (h) and Scale factor along a parallel (k) calculated at one degree interval of latitude at one degree interval of latitude on WGS84.

Table 5 shows the areas where the set limits for area and angular distortions are satisfied. In general areal and angular distortions for Azimuthal projections are considerably lower than those of the Conic and Cylindrical projections and in general satisfy the set limits. A closer look at this table leads to the conclusion that practically unnoticed distortions can be achieved with the Equidistant Azimuthal projection for most part of the Arctic Region. The area between 90° - 71° is covered with maximum area distortion less than 2% and maximum angular distortion 0.95 degrees making this projection the most suitable one. An alternative choice is the Stereographic projection, which behaves comparatively well. Changing the position projection plane will result to lower distortion values for the Stereographic projection. The analysis carried out shows that there is no need for this provided that the limits set are satisfactorily met by the standard form of the Azimuthal projections where the plane is tangent at the pole. On the other hand for the sub Arctic Region the table of distortions [Table 6] directs us to look for other more suitable projection(s) that will portray the area covering latitudes from 50° to 75° with minimum distortions.

Projection	Limits for areal and angular distortions					
	Area: 8 -10%	Angle: 8° - 10°	Area: 6 - 8%	Angle: 6° - 8°	Area: 2%	Angle: 1°
Orthographic	90° - 65°	90° - 58°	90° - 67°	90° - 61°	90° - 79°	90° - 80°
Stereographic	90° - 66°	conformal	90° - 68°	conformal	90° - 79°	conformal
Equal area	equal area	90° - 50°	equal area	90° - 50 ^{o}	equal area	90° - 75°
Equidistant	90° - 50°	90° - 50°	90° - 52°	90° - 50°	90° - 71°	90° - 72°
Gnomonic	90° - 76°	90° - 58°	90° -77°	90° - 61°	90° - 84°	90° - 80°

Table 5. Azimuthal projections: Criteria for angular and areal distortion are satisfied in certain parts of the study area ($\varphi_{min} - \varphi_{max}$).
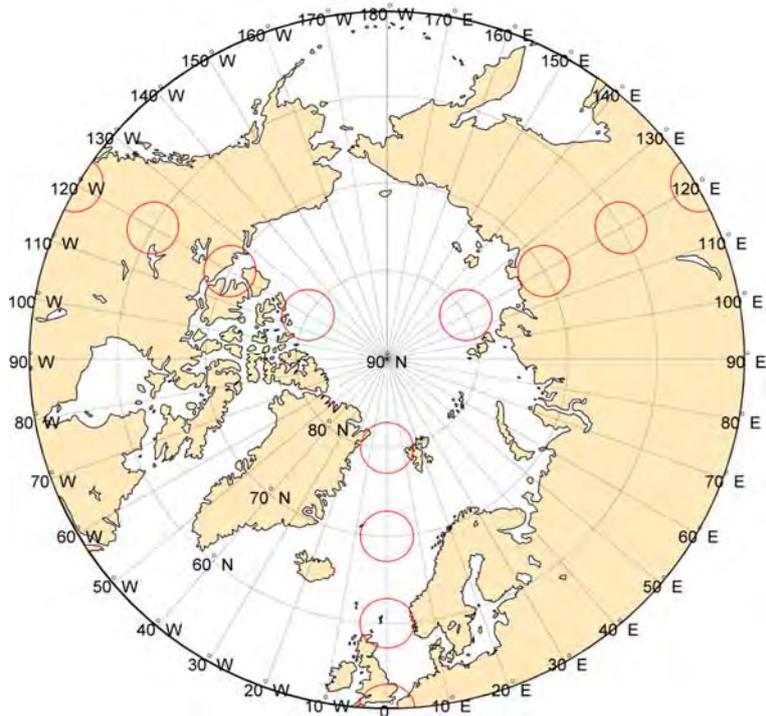
Figure 29. Azimuthal Equal Area projection – Tissot's Ellipses of distortion
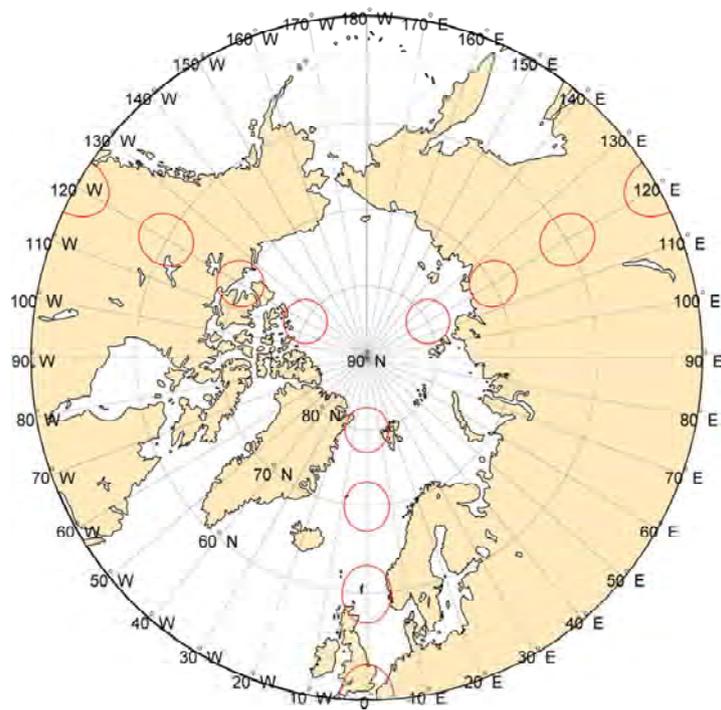
Figure 30. Azimuthal Gnomonic projection – Tissot's Ellipses of distortion
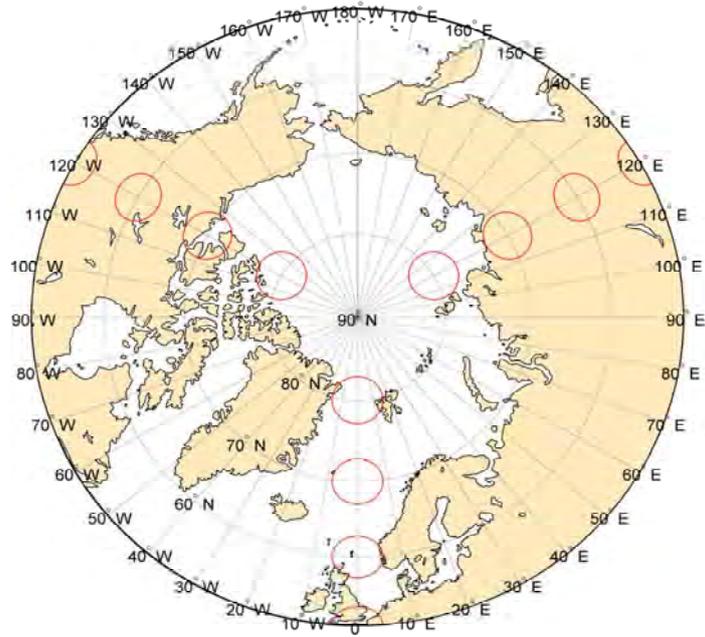
Figure 31. Orthographic projection – Tissot's Ellipses of distortion
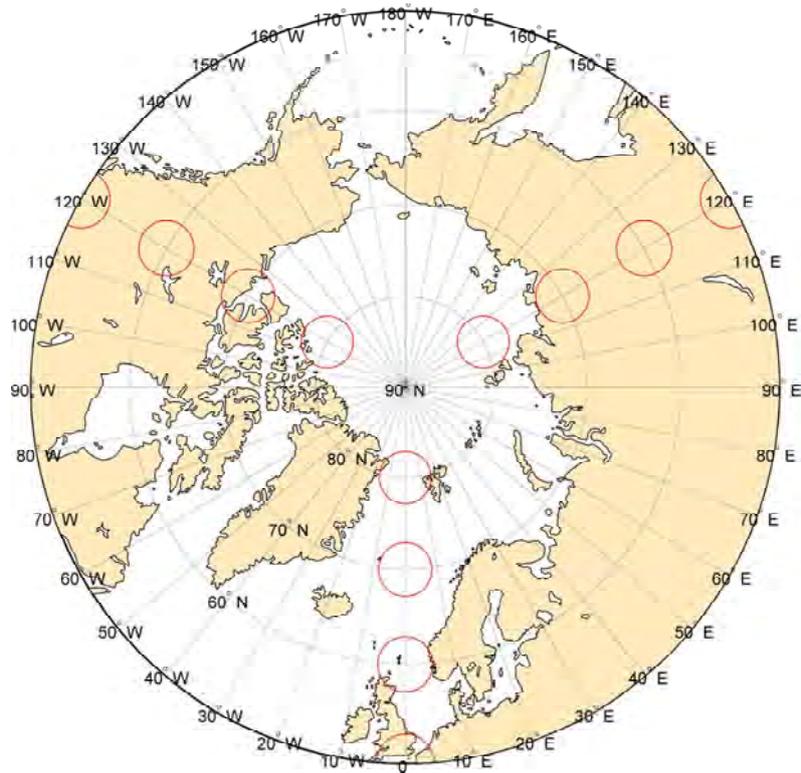
Figure 32. Azimuthal Stereographic projection – Tissot's Ellipses of distortion
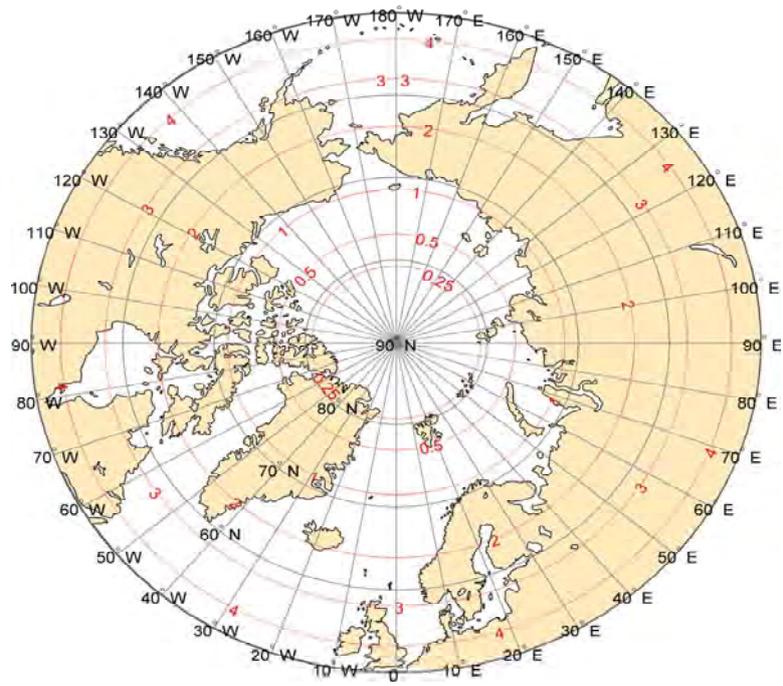
Figure 33. Azimuthal Equidistant projection – Isolines of maximum angular distortion

[0.25, 0.5, 1, 2, 3, 4]
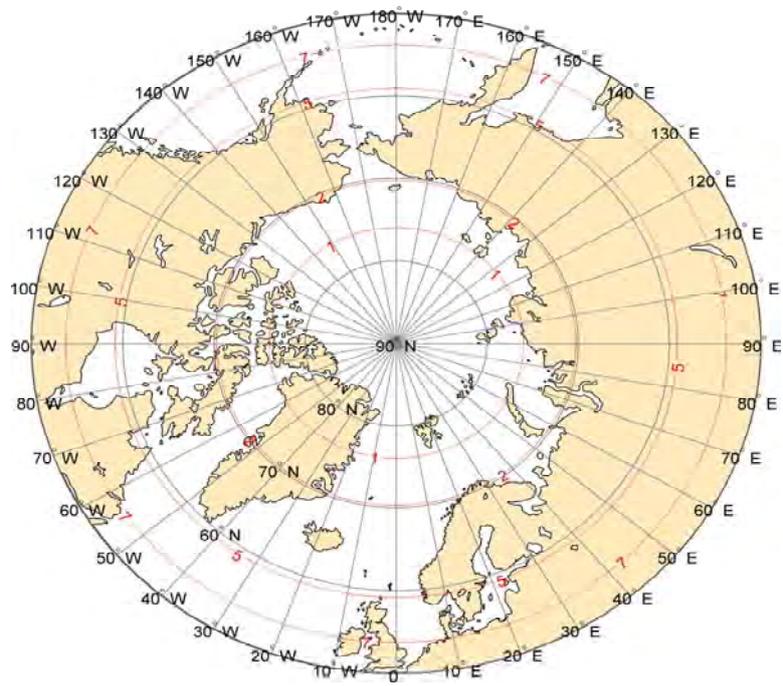
Figure 34. Azimuthal Equidistant projection – Isolines of areal distortion (%) [1, 2, 5, 7]
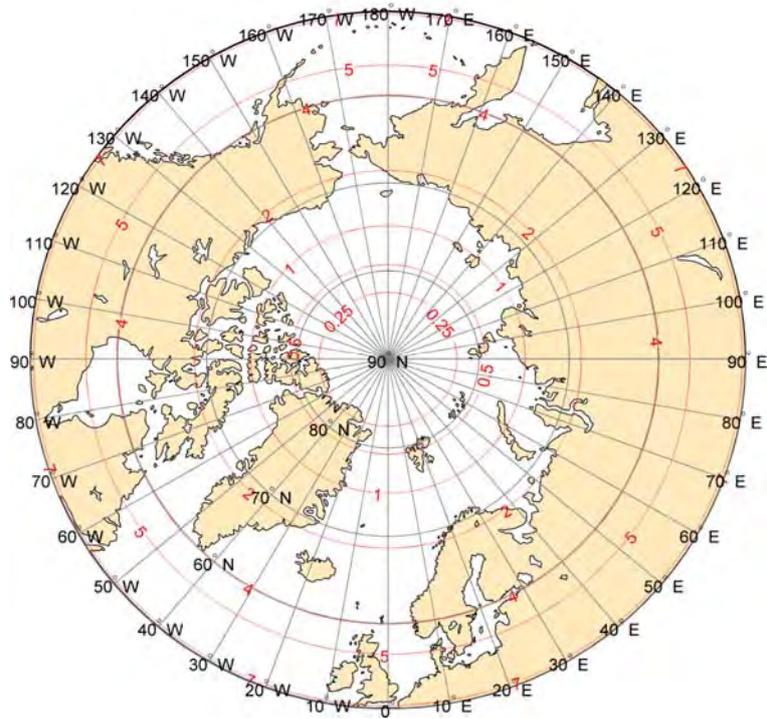
Figure 35. Azimuthal Equal Area projection – Isolines of maximum angular distortion (degrees)
[0.25, 0.5, 1, 2, 4, 5, 7]
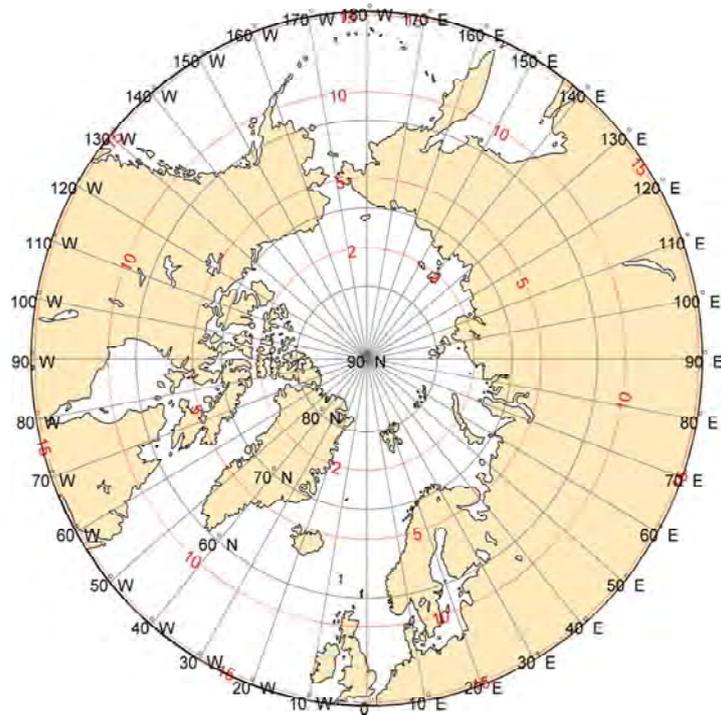
Figure 36. Azimuthal Gnomonic projection – Isolines of maximum angular distortion (degrees)
[2, 5, 10, 15]
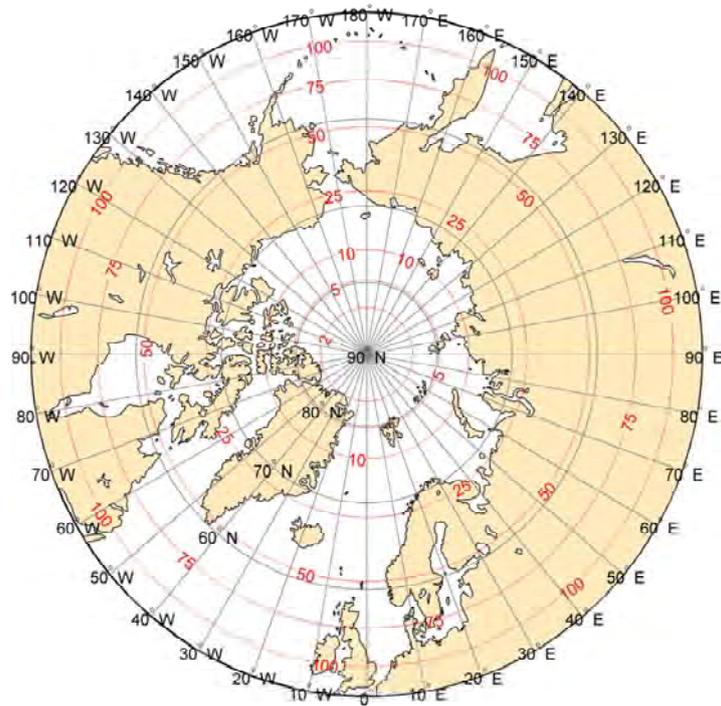
Figure 37. Azimuthal Gnomonic projection – Isolines of areal distortion (%) [2, 5, 10, 25, 50, 75, 100]
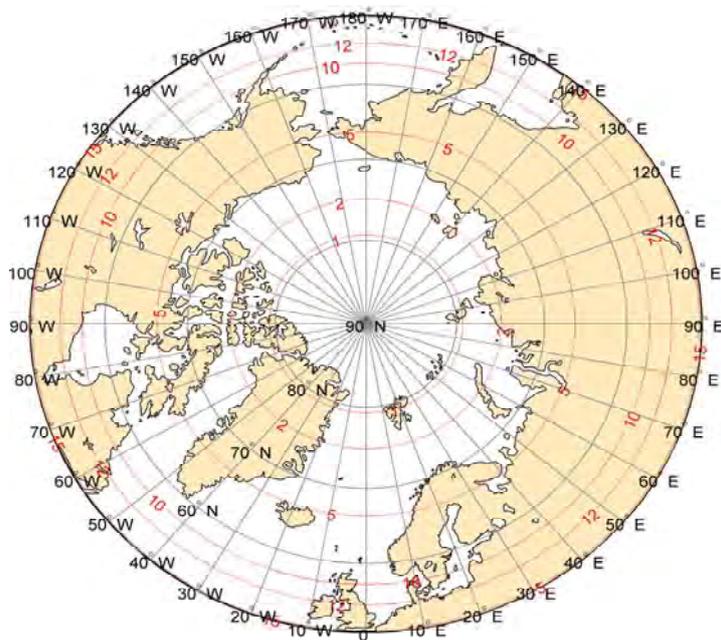
Figure 38. Azimuthal Orthographic projection – Isolines of maximum angular distortion [1, 2, 5, 10, 12]
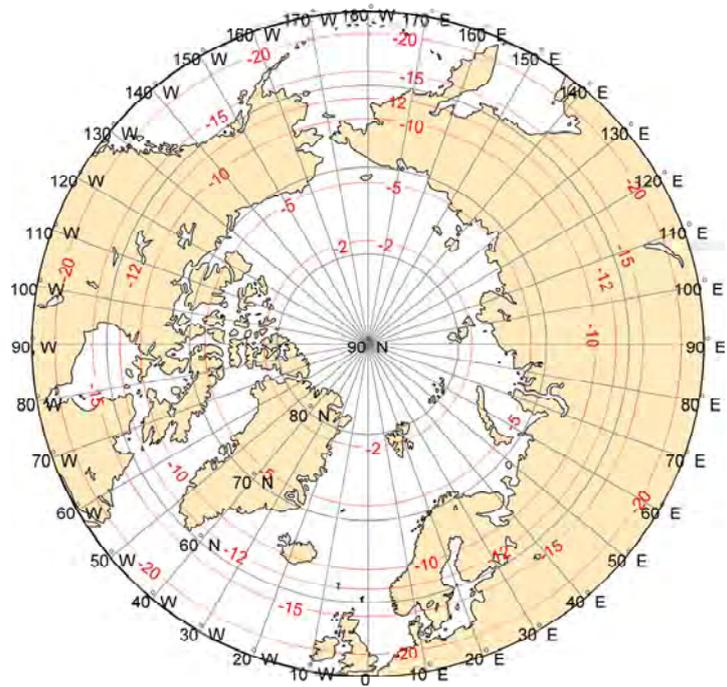
Figure 39. Azimuthal Orthographic projection – Isolines of areal distortion (%) [2, 5, 10, 12, 15, 20]
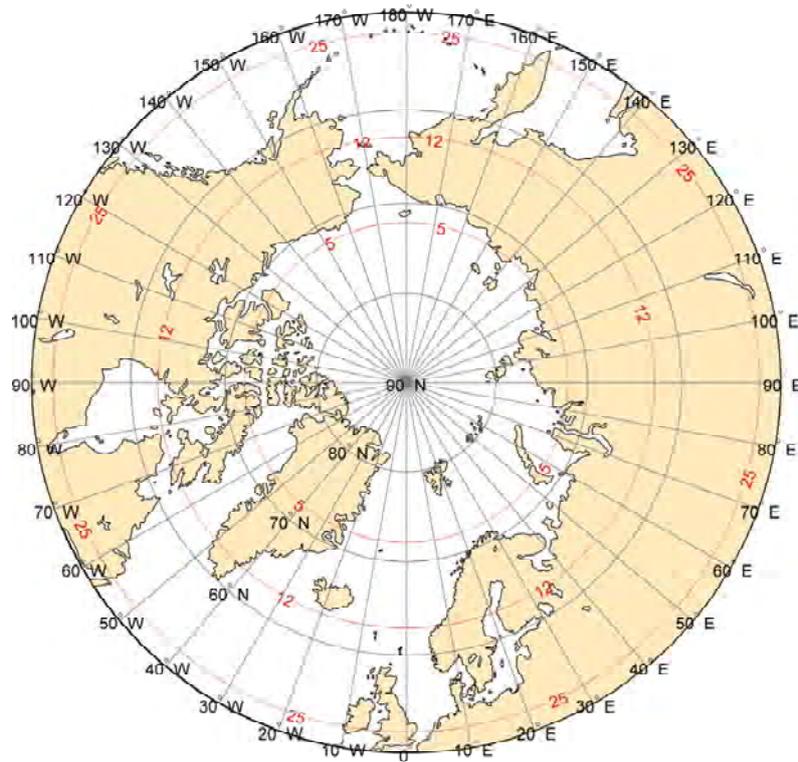
Figure 40. Azimuthal Stereographic projection – Isolines of areal distortion (%) [5, 12, 25]
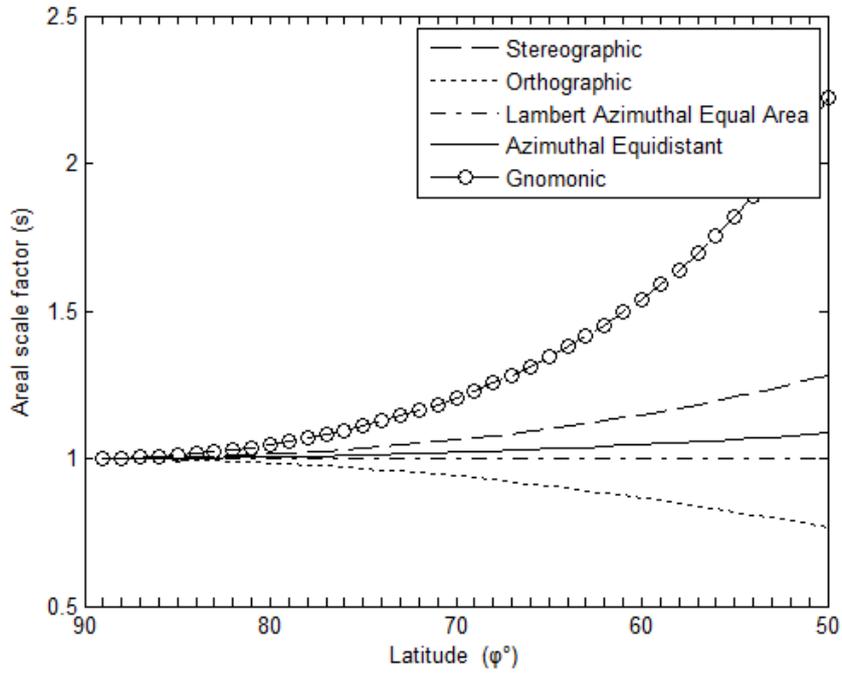
Figure 41. Azimuthal projections: Areal scale factor
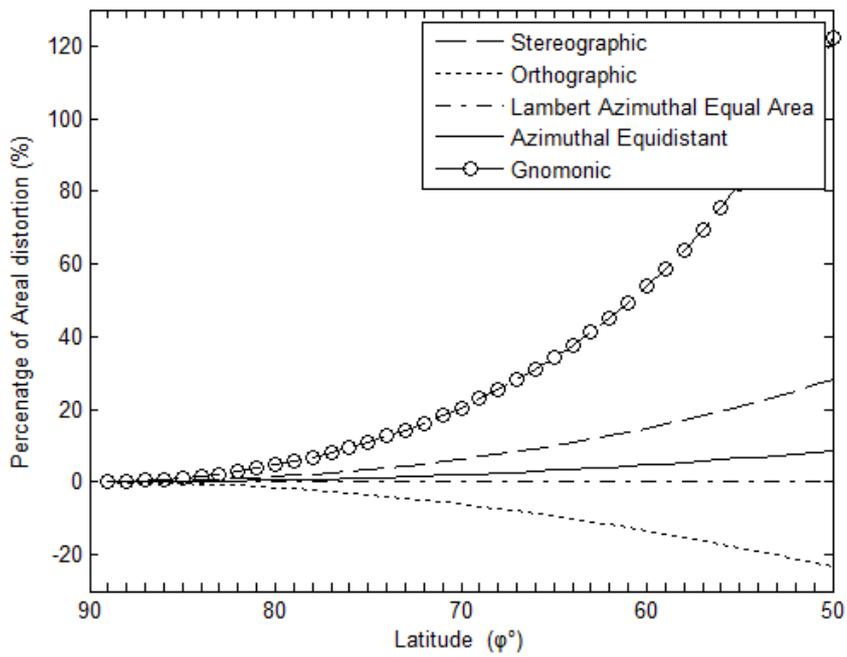
Figure 42. Azimuthal projections: Percentage of areal distortion
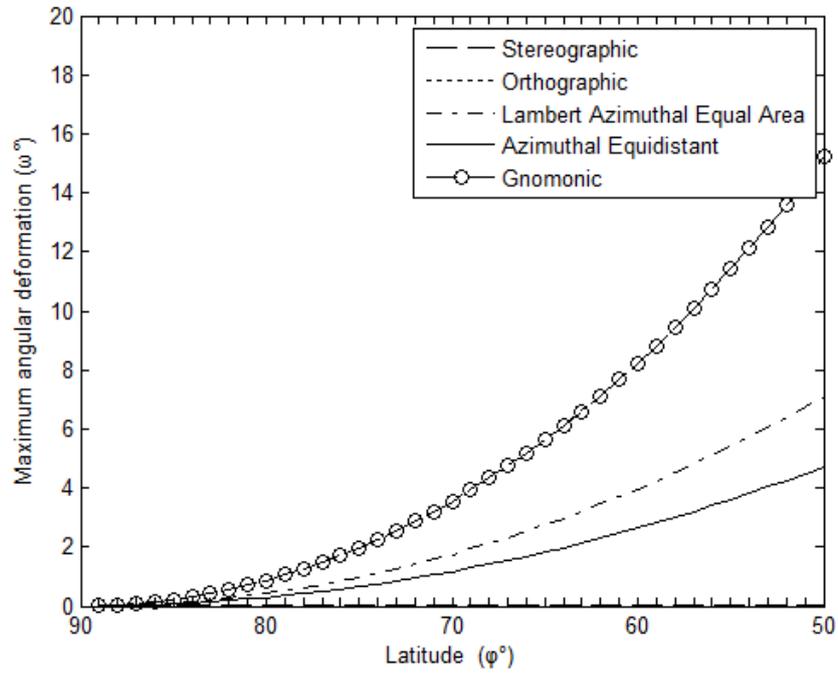
Figure 43. Azimuthal projections: Maximum angular distortion (ω) in degrees
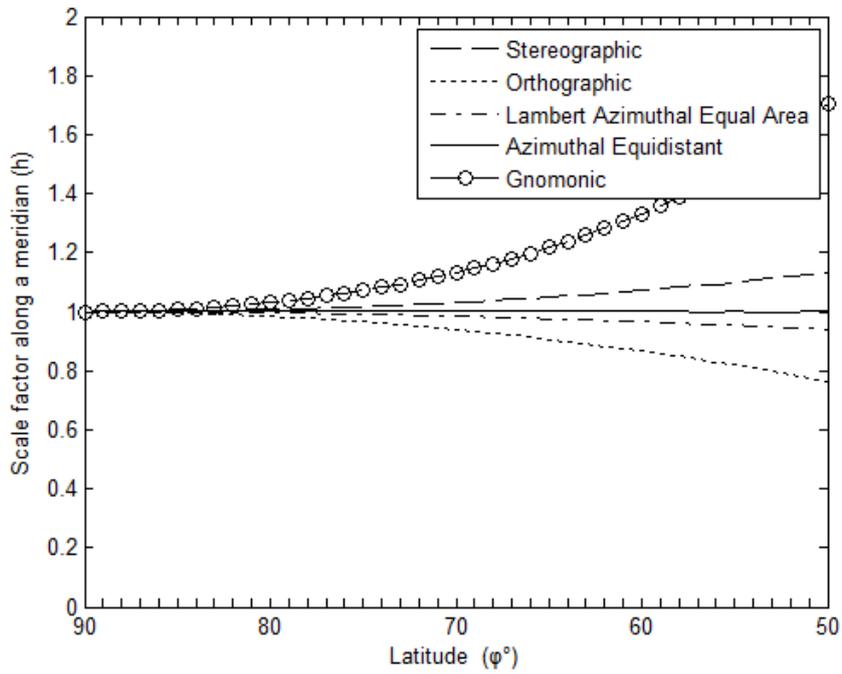
Figure 44. Azimuthal projections: Scale factor along a meridian (h)
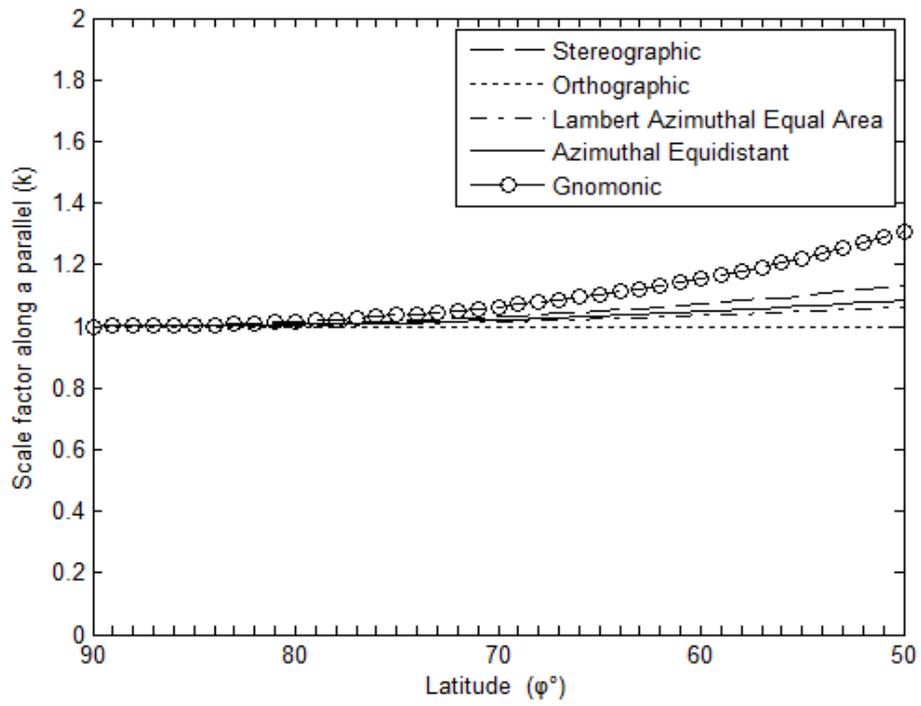
Figure 45. Azimuthal projections: Scale factor along a parallel (k)

φ	ortho area	area %	angles	k	h	stereo area	area %	angles	k	h	equalarea area	area %	angles	k	h	eqdiz area	area %	angles	k	h	gnomonic area	area %	angles	k	h
90	1,00	0,00	0,00	1	1	1,00	0,00	0,00	1,00	1,00	1	0	0	1	1	1	0	0,00	1,00	1	1	0	0	1	1
89	1,00	-0,02	0,01	1	1,00	1,00	0,02	0,00	1,00	1,00	1	0	0,00	1,00	1,00	1,00	0,00	0,00	1,00	1	1,00	0,05	0,01	1,00	1,00
88	1,00	-0,06	0,03	1	1,00	1,00	0,06	0,00	1,00	1,00	1	0	0,02	1,00	1,00	1,00	0,02	0,01	1,00	1	1,00	0,18	0,03	1,00	1,00
87	1,00	-0,14	0,08	1	1,00	1,00	0,14	0,00	1,00	1,00	1	0	0,04	1,00	1,00	1,00	0,04	0,02	1,00	1	1,00	0,41	0,08	1,00	1,00
86	1,00	-0,24	0,14	1	1,00	1,00	0,24	0,00	1,00	1,00	1	0	0,07	1,00	1,00	1,00	0,08	0,05	1,00	1	1,01	0,73	0,14	1,00	1,00
85	1,00	-0,38	0,22	1	1,00	1,00	0,38	0,00	1,00	1,00	1	0	0,11	1,00	1,00	1,00	0,12	0,07	1,00	1	1,01	1,15	0,22	1,00	1,01
84	0,99	-0,55	0,31	1	0,99	1,01	0,55	0,00	1,00	1,00	1	0	0,16	1,00	1,00	1,00	0,18	0,10	1,00	1	1,02	1,66	0,31	1,01	1,01
83	0,99	-0,75	0,43	1	0,99	1,01	0,75	0,00	1,00	1,00	1	0	0,21	1,00	1,00	1,00	0,25	0,14	1,00	1	1,02	2,27	0,43	1,01	1,02
82	0,99	-0,97	0,56	1	0,99	1,01	0,98	0,00	1,00	1,00	1	0	0,28	1,00	1,00	1,00	0,32	0,19	1,00	1	1,03	2,98	0,56	1,01	1,02
81	0,99	-1,23	0,71	1	0,99	1,01	1,24	0,00	1,01	1,01	1	0	0,35	1,00	1,00	1,00	0,41	0,23	1,00	1	1,04	3,79	0,71	1,01	1,03
80	0,98	-1,52	0,88	1	0,98	1,02	1,54	0,00	1,01	1,01	1	0	0,44	1,00	1,00	1,01	0,51	0,29	1,01	1	1,05	4,70	0,88	1,02	1,03
79	0,98	-1,84	1,06	1	0,98	1,02	1,86	0,00	1,01	1,01	1	0	0,53	1,00	1,00	1,01	0,61	0,35	1,01	1	1,06	5,72	1,06	1,02	1,04
78	0,98	-2,18	1,27	1	0,98	1,02	2,22	0,00	1,01	1,01	1	0	0,63	1,01	0,99	1,01	0,73	0,42	1,01	1	1,07	6,85	1,27	1,02	1,05
77	0,97	-2,56	1,49	1	0,97	1,03	2,61	0,00	1,01	1,01	1	0	0,74	1,01	0,99	1,01	0,86	0,49	1,01	1	1,08	8,10	1,49	1,03	1,05
76	0,97	-2,97	1,73	1	0,97	1,03	3,04	0,00	1,02	1,02	1	0	0,86	1,01	0,99	1,01	1,00	0,57	1,01	1	1,09	9,46	1,73	1,03	1,06
75	0,97	-3,41	1,99	1	0,97	1,03	3,50	0,00	1,02	1,02	1	0	0,98	1,01	0,99	1,01	1,15	0,65	1,01	1	1,11	10,96	1,99	1,04	1,07
74	0,96	-3,87	2,26	1	0,96	1,04	3,99	0,00	1,02	1,02	1	0	1,12	1,01	0,99	1,01	1,31	0,75	1,01	1	1,13	12,58	2,26	1,04	1,08
73	0,96	-4,37	2,56	1	0,96	1,05	4,52	0,00	1,02	1,02	1	0	1,27	1,01	0,99	1,01	1,48	0,84	1,01	1	1,14	14,33	2,56	1,05	1,09
72	0,95	-4,89	2,87	1	0,95	1,05	5,08	0,00	1,03	1,03	1	0	1,42	1,01	0,99	1,02	1,66	0,94	1,02	1	1,16	16,24	2,87	1,05	1,11
71	0,95	-5,44	3,21	1	0,95	1,06	5,68	0,00	1,03	1,03	1	0	1,58	1,01	0,99	1,02	1,85	1,05	1,02	1	1,18	18,29	3,21	1,06	1,12
70	0,94	-6,03	3,56	1	0,94	1,06	6,31	0,00	1,03	1,03	1	0	1,75	1,02	0,98	1,02	2,06	1,17	1,02	1	1,20	20,50	3,56	1,06	1,13
69	0,93	-6,64	3,93	1	0,93	1,07	6,98	0,00	1,03	1,03	1	0	1,93	1,02	0,98	1,02	2,27	1,29	1,02	1	1,23	22,88	3,93	1,07	1,15
68	0,93	-7,27	4,33	1	0,93	1,08	7,70	0,00	1,04	1,04	1	0	2,12	1,02	0,98	1,02	2,50	1,41	1,02	1	1,25	25,43	4,33	1,08	1,16
67	0,92	-7,94	4,74	1	0,92	1,08	8,45	0,00	1,04	1,04	1	0	2,32	1,02	0,98	1,03	2,73	1,55	1,03	1	1,28	28,18	4,74	1,09	1,18
66	0,91	-8,64	5,17	1	0,91	1,09	9,23	0,00	1,05	1,05	1	0	2,53	1,02	0,98	1,03	2,98	1,68	1,03	1	1,31	31,12	5,17	1,09	1,20
65	0,91	-9,36	5,63	1	0,91	1,10	10,06	0,00	1,05	1,05	1	0	2,75	1,02	0,98	1,03	3,24	1,83	1,03	1	1,34	34,28	5,63	1,10	1,22
64	0,90	-10,11	6,10	1	0,90	1,11	10,94	0,00	1,05	1,05	1	0	2,97	1,03	0,97	1,04	3,51	1,98	1,04	1	1,38	37,67	6,10	1,11	1,24
63	0,89	-10,88	6,60	1	0,89	1,12	11,85	0,00	1,06	1,06	1	0	3,21	1,03	0,97	1,04	3,80	2,13	1,04	1	1,41	41,30	6,60	1,12	1,26
62	0,88	-11,69	7,12	1	0,88	1,13	12,81	0,00	1,06	1,06	1	0	3,45	1,03	0,97	1,04	4,09	2,30	1,04	1	1,45	45,19	7,12	1,13	1,28
61	0,87	-12,52	7,66	1	0,87	1,14	13,81	0,00	1,07	1,07	1	0	3,71	1,03	0,97	1,04	4,40	2,47	1,04	1	1,49	49,36	7,66	1,14	1,31
60	0,87	-13,38	8,22	1	0,87	1,15	14,86	0,00	1,07	1,07	1	0	3,97	1,04	0,97	1,05	4,72	2,64	1,05	1	1,54	53,84	8,22	1,15	1,33
59	0,86	-14,26	8,80	1	0,86	1,16	15,96	0,00	1,08	1,08	1	0	4,24	1,04	0,96	1,05	5,05	2,82	1,05	1	1,59	58,64	8,80	1,17	1,36
58	0,85	-15,17	9,41	1	0,85	1,17	17,10	0,00	1,08	1,08	1	0	4,52	1,04	0,96	1,05	5,39	3,01	1,05	1	1,64	63,79	9,41	1,18	1,39
57	0,84	-16,10	10,05	1	0,84	1,18	18,30	0,00	1,09	1,09	1	0	4,81	1,04	0,96	1,06	5,74	3,20	1,06	1	1,69	69,33	10,05	1,19	1,42
56	0,83	-17,06	10,70	1	0,83	1,20	19,54	0,00	1,09	1,09	1	0	5,11	1,05	0,96	1,06	6,11	3,40	1,06	1	1,75	75,27	10,70	1,21	1,45
55	0,82	-18,04	11,38	1	0,82	1,21	20,84	0,00	1,10	1,10	1	0	5,42	1,05	0,95	1,06	6,49	3,60	1,06	1	1,82	81,67	11,38	1,22	1,49
54	0,81	-19,05	12,09	1	0,81	1,22	22,20	0,00	1,11	1,11	1	0	5,74	1,05	0,95	1,07	6,89	3,82	1,07	1	1,89	88,55	12,09	1,24	1,53
53	0,80	-20,09	12,82	1	0,80	1,24	23,61	0,00	1,11	1,11	1	0	6,07	1,05	0,95	1,07	7,30	4,03	1,07	1	1,96	95,96	12,82	1,25	1,57
52	0,79	-21,15	13,58	1	0,79	1,25	25,08	0,00	1,12	1,12	1	0	6,41	1,06	0,95	1,08	7,72	4,26	1,08	1	2,04	103,95	13,58	1,27	1,61
51	0,78	-22,23	14,36	1	0,78	1,27	26,61	0,00	1,13	1,13	1	0	6,76	1,06	0,94	1,08	8,15	4,49	1,08	1	2,13	112,57	14,36	1,29	1,65
50	0,77	-23,50	15,30	1	0,77	1,28	28,20	0,00	1,13	1,13	1	0	7,12	1,06	0,94	1,09	8,60	4,73	1,09	1	2,23	123,33	15,30	1,31	1,71

Table 6. Azimuthal projections: Values for area distortion, Percentage of areal distortion, Maximum angular deformation (ω) in decimal degrees, Scale factor along a parallel (k), Scale factor along a meridian (h) for Orthographic, Stereographic, Equal Area Azimuthal, Equidistant Azimuthal and Gnomonic projections.

2.4 Study of map projections for the sub-Arctic Regions

The results of the analysis of the distortions for the projections studied lead to the necessity of addressing the issue of projection selection separately for the sub Arctic Region ($50^o – 75^o$). For this part of the study Conic and Cylindrical projections are analyzed.

2.4.1 Conic Projections

Three Conic projections are studied: Lambert Conformal, Conic Equal Area, and Conic Equidistant. In order to minimize the distortion two standard parallels are selected as discussed earlier at 55° and 70° for all three projections.

The study of projections and the evaluation of the distortions are based on:

- Maps portraying Tissot's ellipses of distortion: Figures 46 - 48
- Maps portraying isolines of angular and areal distortion: Figures 49 - 52
- Diagrams of Areal scale factor, Percentage of Areal distortion, Maximum angular distortion (ω), Scale factor along a meridian (h) and Scale factor along a parallel (k) calculated at one degree interval of latitude: Figures 53 - 58
- Table 8 showing values of Areal scale factor, Percentage of Areal distortion, Maximum angular distortion (ω), Scale factor along a meridian (h) and Scale factor along a parallel (k) calculated at one degree interval of latitude at one degree interval of latitude on WGS84

The following table shows the areas where the set limits for area and angular distortions are satisfied. In general areal and angular distortions for Conic projections satisfy the set limits with the Lambert Conformal and Conic Equidistant projections giving the best results.

Projection	Limits for angular and areal distortions					
	Area: 8 -10%	Angle: 8° -10°	Area: 6 - 8%	Angle: 6° - 8°	Area: 2%	Angle: 1°
Lambert	50° - 75°	conformal	50° - 75°	conformal	52° - 73°	conformal
Conic Equal Area	equal area	50° - 75°	equal area	50° - 75°	equal area	51° - 72°
Conic Equidistant	50° - 75°	50° - 75°	50° - 75°	50° - 75°	50° - 74°	50° - 74°

Table 7. Conic projections: Criteria for angular and areal distortions are satisfied in certain parts of the study area ($\varphi_{min} – \varphi_{max}$).
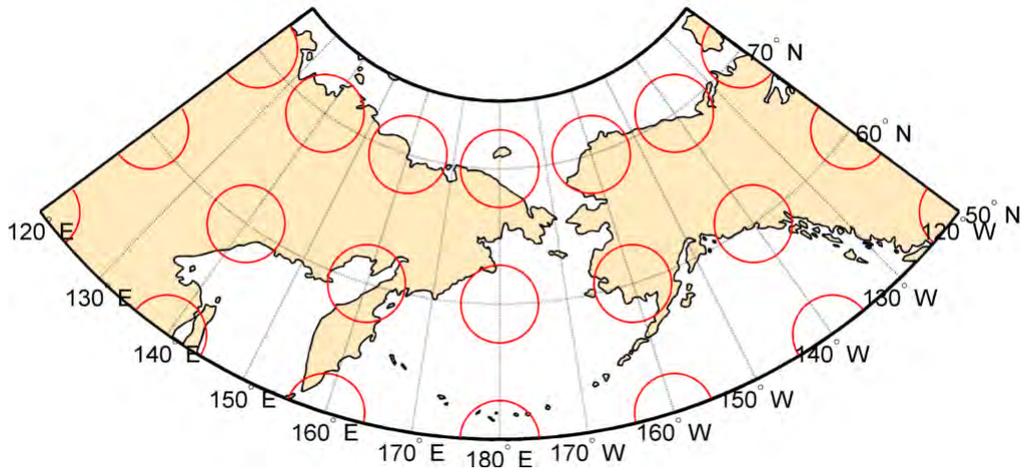
Figure 46. Conic Equidistant projection – Tissot's Ellipses of distortion
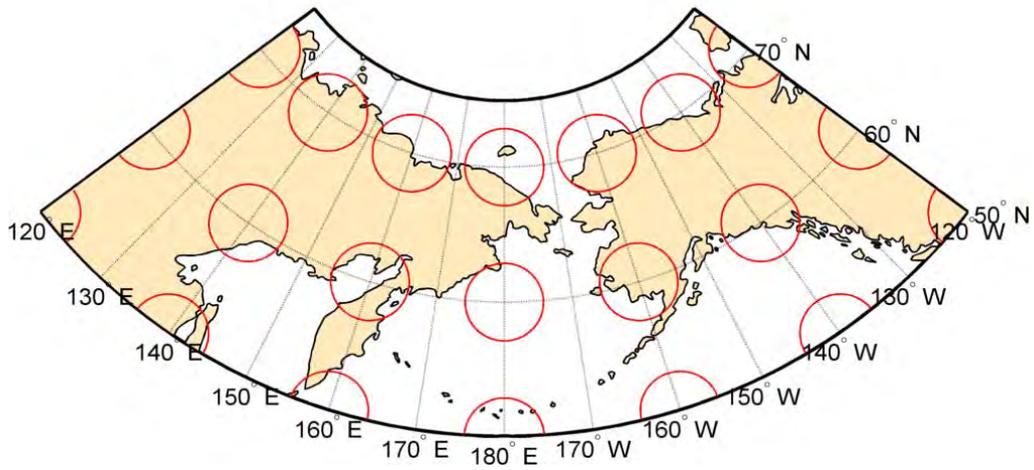
Figure 47. Conic Equal Area projection – Tissot's Ellipses of distortion
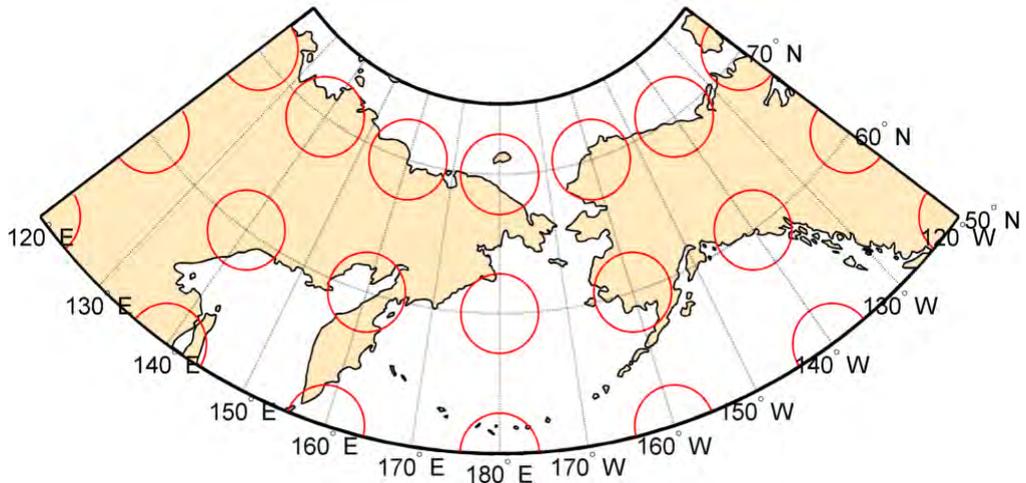
Figure 48. Lambert Conformal projection – Tissot's Ellipses of distortion
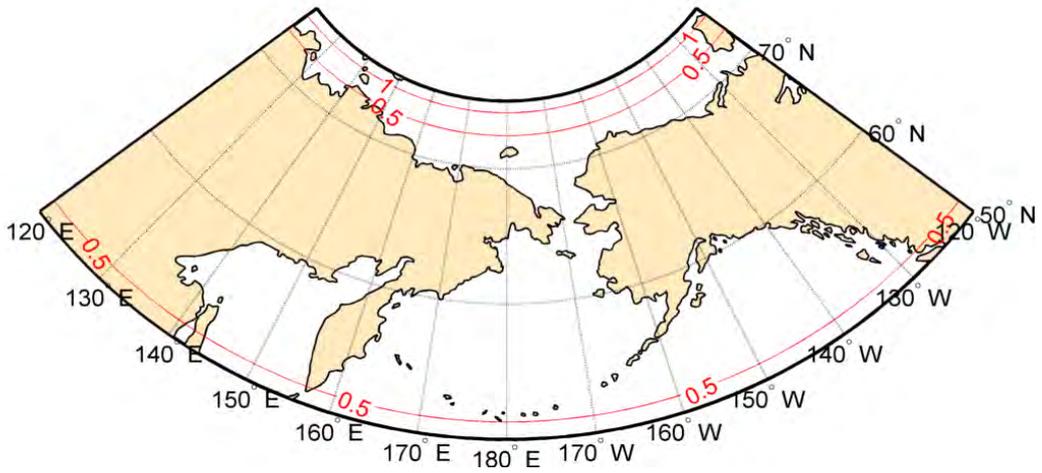
Figure 49. Conic Equidistant. projection – Isolines of maximum angular distortion (degrees) [0.5, 1]
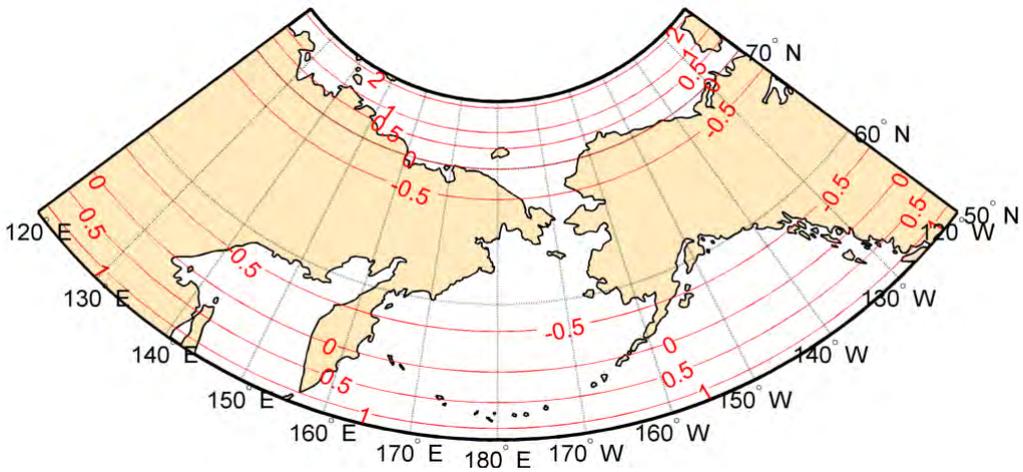

Figure 50. Conic Equidistant projection -- Isolines of area distortion (%) [0, 0.5, 1, 2]
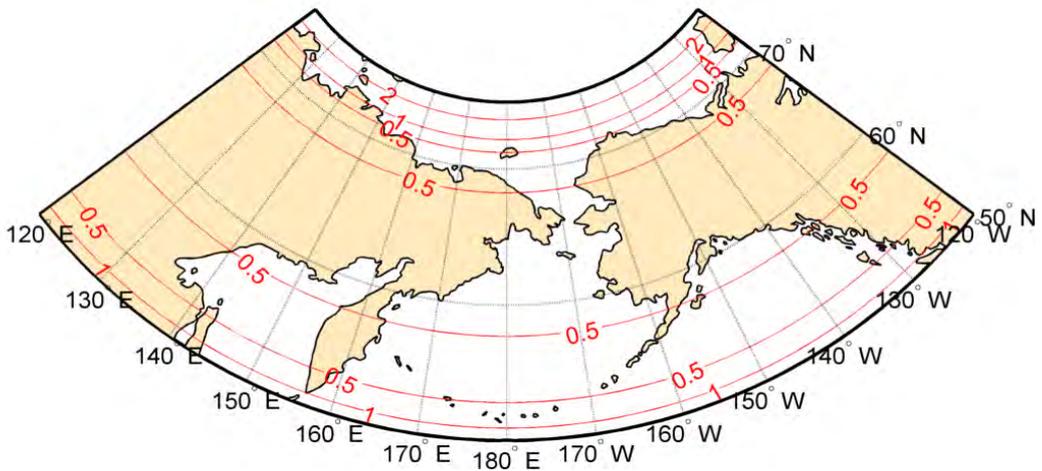
Figure 51. Conic Equal Area projection – Isolines of maximum angular distortion (degrees) [0.5, 1, 2]
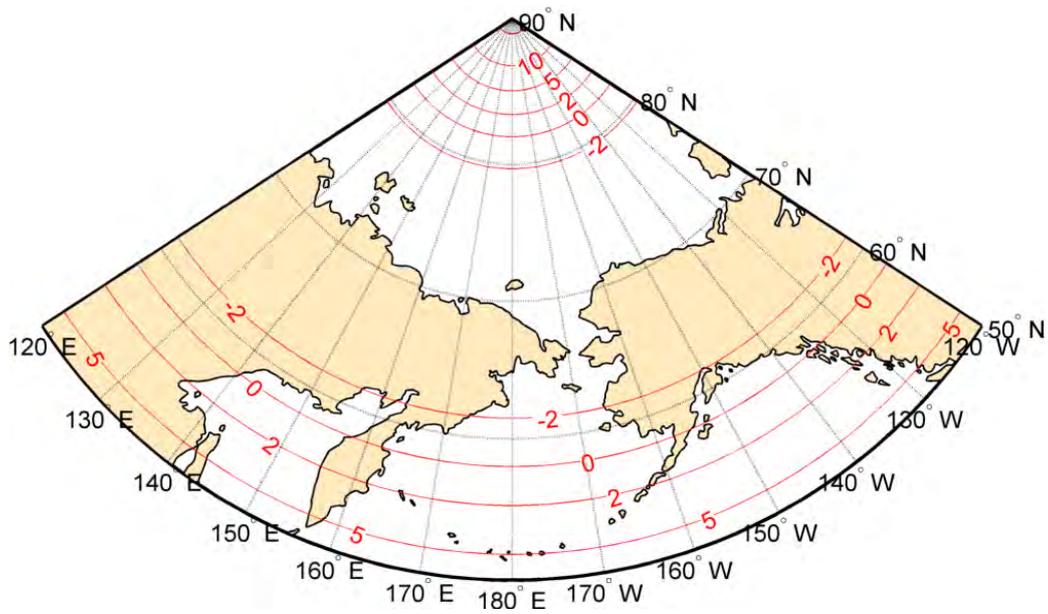
Figure 52. Lambert Conformal projection - Isolines of area distortion (%) [0 2, 5, 10)
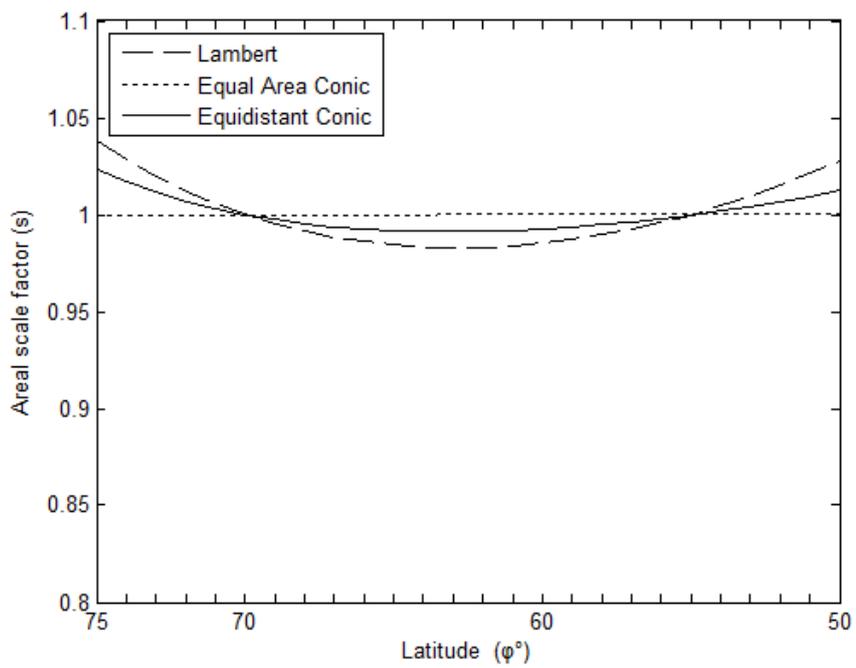
Figure 53. Conic projections (sub-Arctic area): Area scale factor
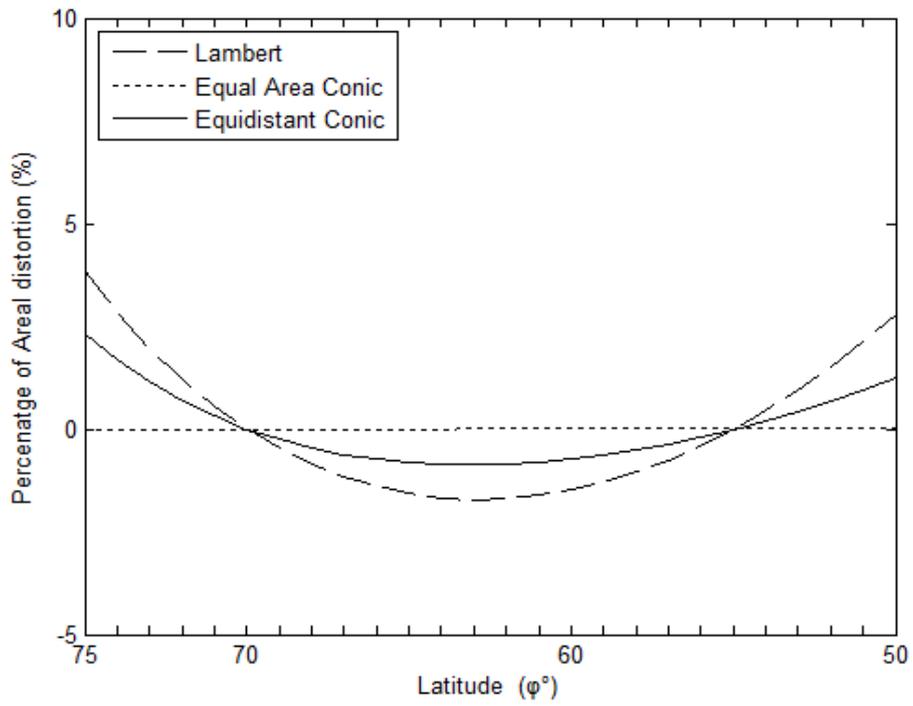
Figure 54. Conic projections (sub-Arctic area): Percentage of area distortion (%)
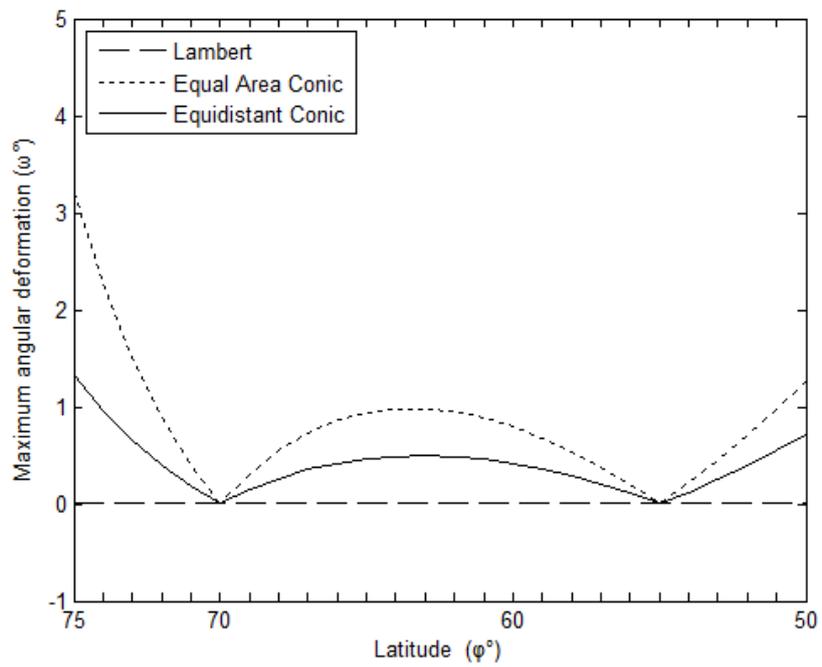
Figure 55. Conic Projections (sub-Arctic area): Maximum angular distortion in degrees

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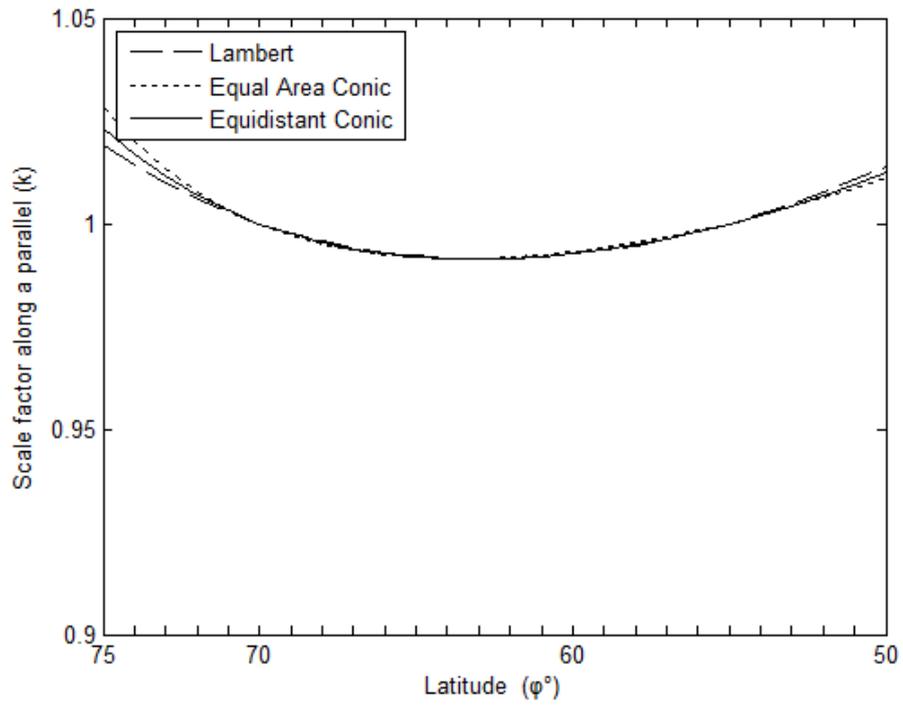
Figure 56. Conic Projections (sub-Arctic area): Scale factor along parallel (k)
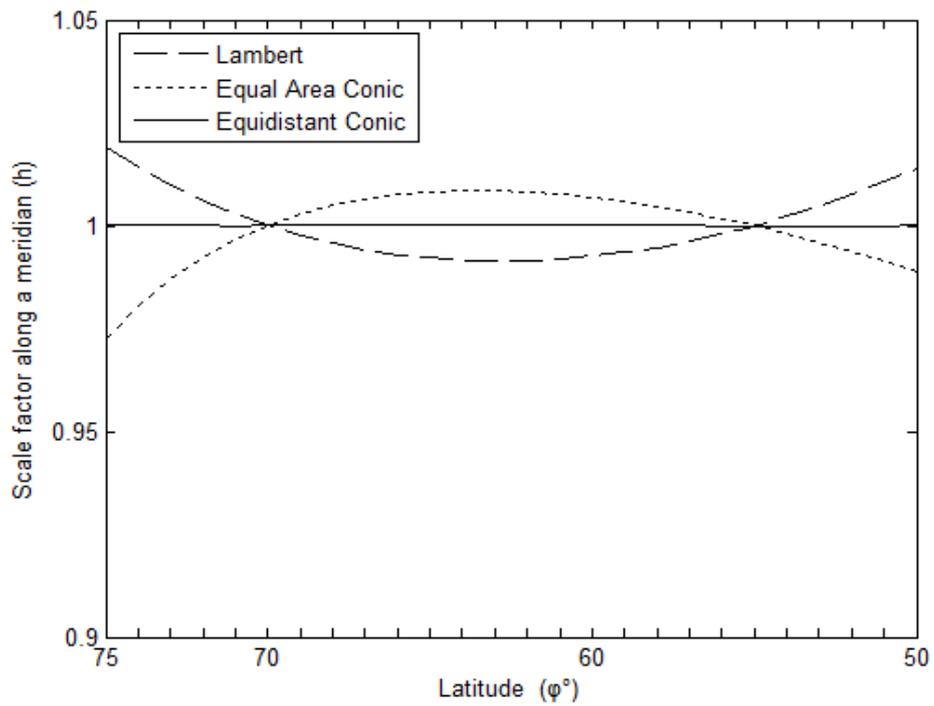
Figure 57. Conic Projections (sub-Arctic area): Scale factor along meridian (h)
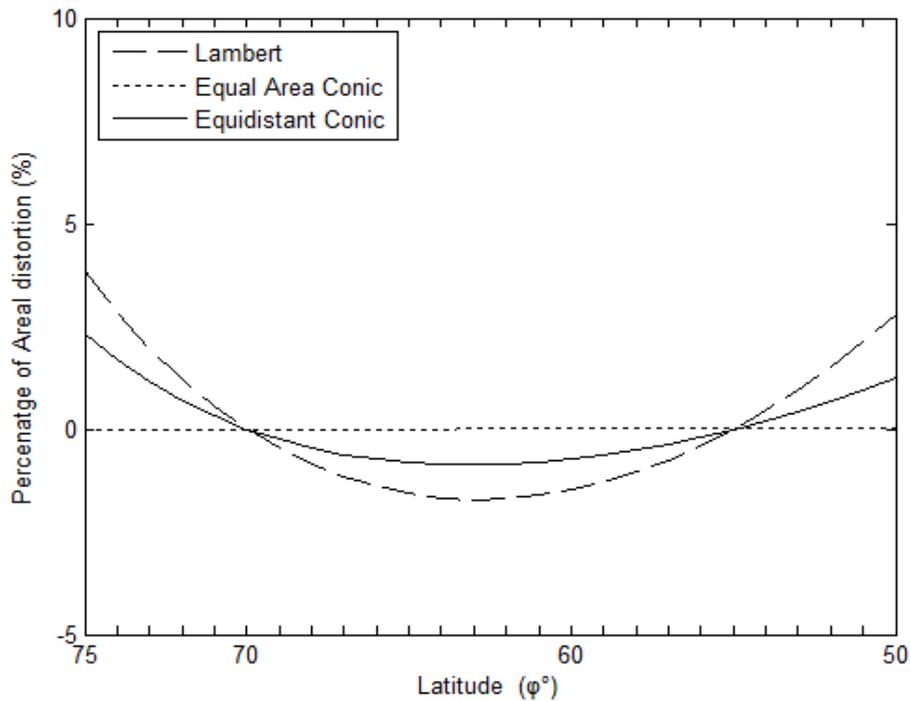
Figure 58. Conic projections (sub-Arctic area): Percentage of area distortion (%)

φ	lambert					equalarea					eqdiz				
	area	area %	angles	k	h	area	area %	angles	k	h	area	area %	angles	k	h
75	1,04	3,83	0,00	1,02	1,02	1	0	3,20	1,03	0,97	1,02	2,32	1,31	1,02	1
74	1,03	2,84	0,00	1,01	1,01	1	0	2,26	1,02	0,98	1,02	1,68	0,95	1,02	1
73	1,02	1,96	0,00	1,01	1,01	1	0	1,50	1,01	0,99	1,01	1,14	0,65	1,01	1
72	1,01	1,21	0,00	1,01	1,01	1	0	0,89	1,01	0,99	1,01	0,69	0,39	1,01	1
71	1,01	0,56	0,00	1,00	1,00	1	0	0,39	1,00	1,00	1,00	0,31	0,18	1,00	1
70	1	0	0	1	1	1	0	0	1	1	1	0	0	1	1
69	1,00	-0,46	0,00	1,00	1,00	1	0	0,31	1,00	1,00	1,00	-0,25	0,15	1,00	1
68	0,99	-0,85	0,00	1,00	1,00	1	0	0,56	1,00	1,00	1,00	-0,45	0,26	1,00	1
67	0,99	-1,16	0,00	0,99	0,99	1	0	0,74	0,99	1,01	0,99	-0,61	0,35	0,99	1
66	0,99	-1,40	0,00	0,99	0,99	1	0	0,86	0,99	1,01	0,99	-0,73	0,42	0,99	1
65	0,98	-1,57	0,00	0,99	0,99	1	0	0,94	0,99	1,01	0,99	-0,80	0,47	0,99	1
64	0,98	-1,67	0,00	0,99	0,99	1	0	0,98	0,99	1,01	0,99	-0,85	0,49	0,99	1
63	0,98	-1,71	0,00	0,99	0,99	1	0	0,98	0,99	1,01	0,99	-0,86	0,50	0,99	1
62	0,98	-1,69	0,00	0,99	0,99	1	0	0,95	0,99	1,01	0,99	-0,84	0,49	0,99	1
61	0,98	-1,61	0,00	0,99	0,99	1	0	0,89	0,99	1,01	0,99	-0,79	0,46	0,99	1
60	0,99	-1,48	0,00	0,99	0,99	1	0	0,80	0,99	1,01	0,99	-0,72	0,42	0,99	1
59	0,99	-1,29	0,00	0,99	0,99	1	0	0,68	0,99	1,01	0,99	-0,62	0,36	0,99	1
58	0,99	-1,05	0,00	0,99	0,99	1	0	0,54	1,00	1,00	1,00	-0,50	0,29	1,00	1
57	0,99	-0,75	0,00	1,00	1,00	1	0	0,38	1,00	1,00	1,00	-0,36	0,20	1,00	1
56	1,00	-0,40	0,00	1,00	1,00	1	0	0,20	1,00	1,00	1,00	-0,19	0,11	1,00	1
55	1	0	0	1	1	1	0	0	1	1	1	0	0	1	1
54	1,00	0,45	0,00	1,00	1,00	1	0	0,22	1,00	1,00	1,00	0,21	0,12	1,00	1
53	1,01	0,96	0,00	1,00	1,00	1	0	0,46	1,00	1,00	1,00	0,43	0,25	1,00	1
52	1,02	1,51	0,00	1,01	1,01	1	0	0,71	1,01	0,99	1,01	0,68	0,39	1,01	1
51	1,02	2,12	0,00	1,01	1,01	1	0	0,98	1,01	0,99	1,01	0,95	0,54	1,01	1
50	1,03	2,78	0,00	1,01	1,01	1	0	1,26	1,01	0,99	1,01	1,23	0,71	1,01	1

Table 8. Conic projections: Values for Area Scale Error, Percentage of area distortion, Maximum angular deformation (ω), Scale factor along a parallel (k), Scale factor along a meridian (k) for Lambert Conformal, Conic Equal Area and Conic Equidistant projections.

2.4.2 Cylindrical Projections

In this section, the use of Cylindrical projections for the sub-Arctic Region is examined with the study of three projections: Mercator (conformal), Cylindrical Equidistant and Cylindrical Equal Area. A standard parallel at $60°$ is selected for all projections. Table 9 shows the areas where the set limits for area and angular distortions are satisfied. In general areal and angular distortions for these projections exceed the set limits that make them visually unobservable and they are satisfied only in small parts of the sub Arctic Region.

The study of projections and the evaluation of the distortions are based on:

- Maps portraying Tissot's ellipses of distortion: Figures 59 - 61
- Maps portraying isolines of angular and areal distortion: Figures 62 - 65
- Diagrams of Areal scale factor, Percentage of Areal distortion, Maximum angular distortion (ω), Scale factor along a meridian (h) and Scale factor along a parallel (k) calculated at one degree interval of latitude: Figures 66 - 69
- Table 10 showing values of Areal scale factor, Percentage of Areal distortion, Maximum angular distortion (ω), Scale factor along a meridian (h) and Scale factor along a parallel (k) calculated at one degree interval of latitude at one degree interval of latitude on WGS84

Concluding and specifically for the sub Arctic Region, the Lambert Conformal and Conic Equidistant projections are those satisfying the set limits of distortions.

Projection	Limits for areal and angular distortion					
	Area: 8 - 10%	Angle: $8° - 10°$	Area: 6 - 8%	Angle: $6° - 8°$	Area: 1%	Angle: $2°$
Mercator	$60° - 61°$	conformal	$60° - 61°$	conformal	$60°$	conformal
Equidistant cylindrical	$57° - 62°$	$54° - 65°$	$62° - 57°$	$55° - 65°$	$60°$	$59° - 61°$
Equal area cylindrical	equal area	$57° - 62°$	equal area	$58° - 62°$	equal area	$60°$

Table 9. Cylindrical Projections: Criteria for angular and areal distortion are satisfied in certain parts of the study area ($\varphi_{min} - \varphi_{max}$).
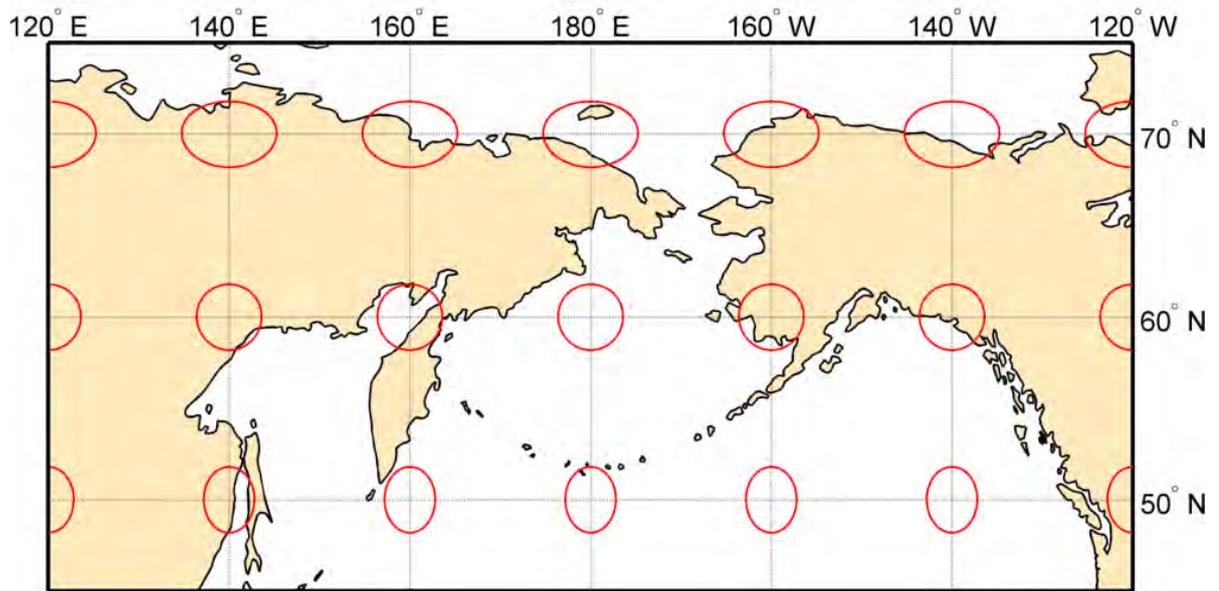
Figure 59. Cylindrical Equidistant projection – Tissot's Ellipses of distortion
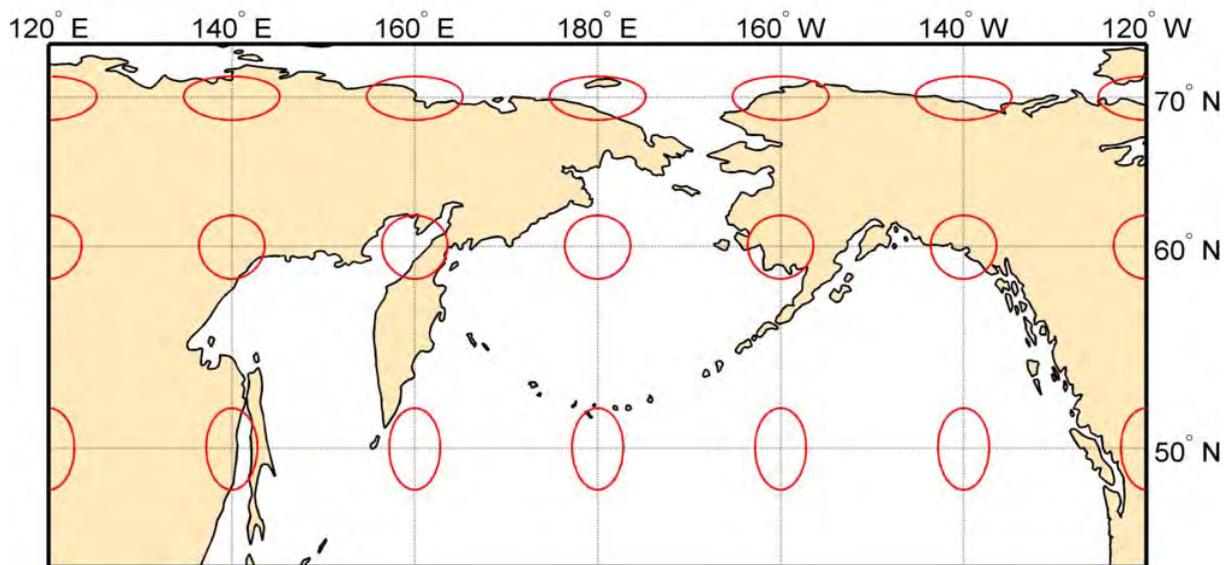
Figure 60. Cylindrical Equal Area projection - Tissot's Ellipses of distortion
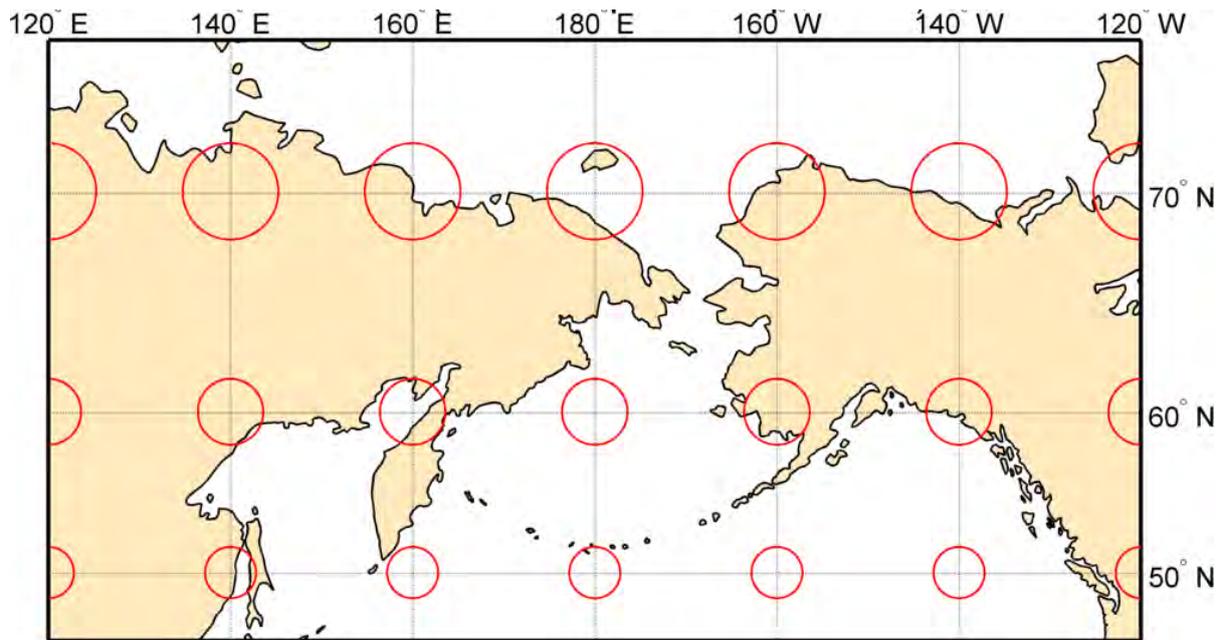
Figure 61. Mercator projection – Tissot's Ellipses of distortion
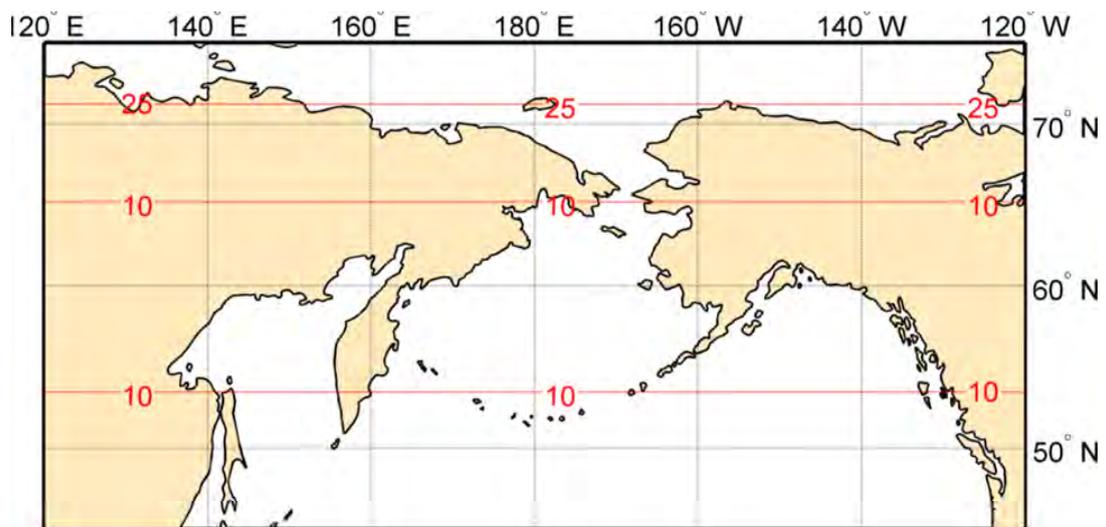
Figure 62. Cylindrical Equidistant projection – Isolines of maximum angular distortion (10, 25 degrees)
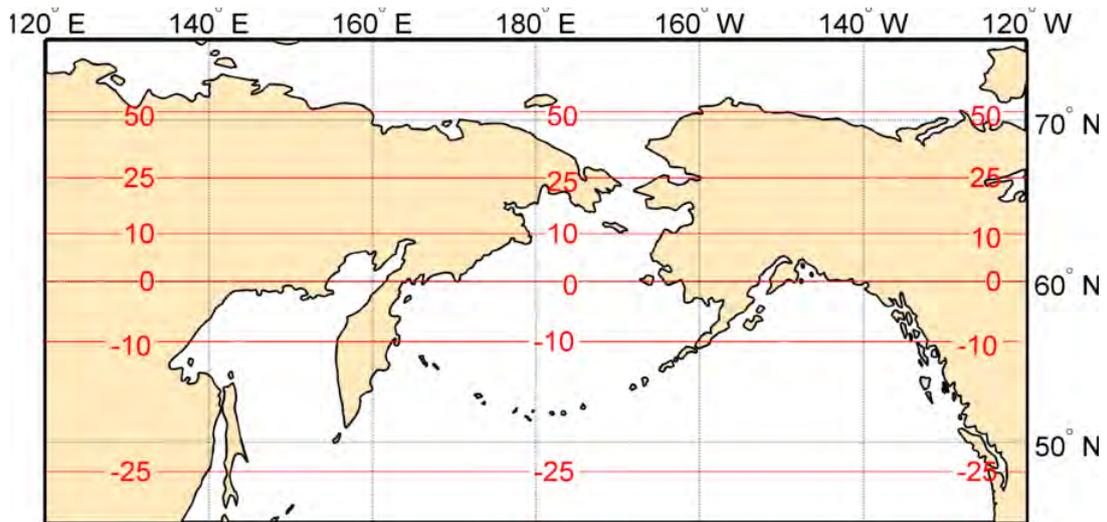
Figure 63. Cylindrical Equidistant projection – Isolines of areal distortion (%) [-25,-10, 0, 25, 50]
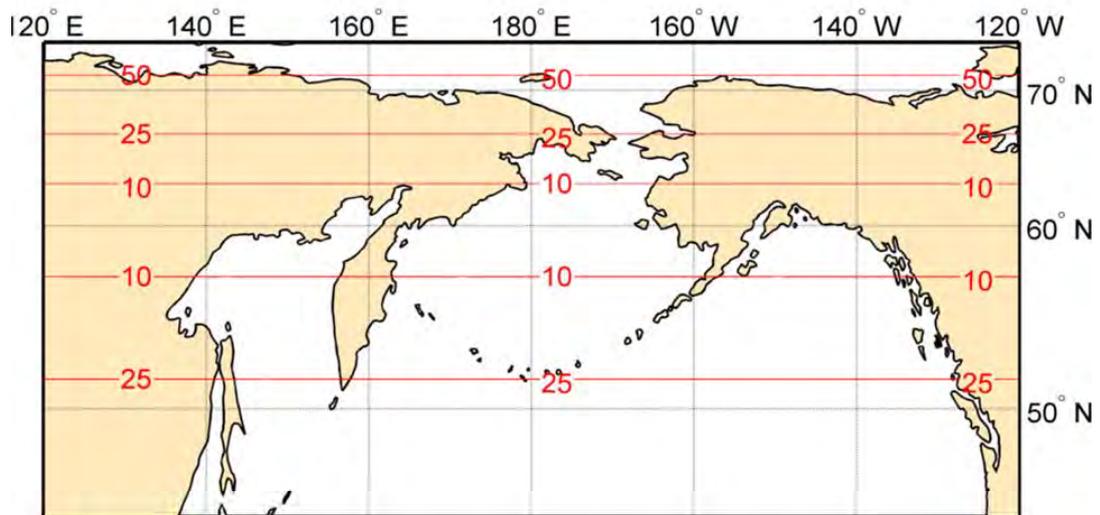
Figure 64. Cylindrical Equal Area projection – Isolines of maximum angular distortion (degrees) [10, 25, 50]
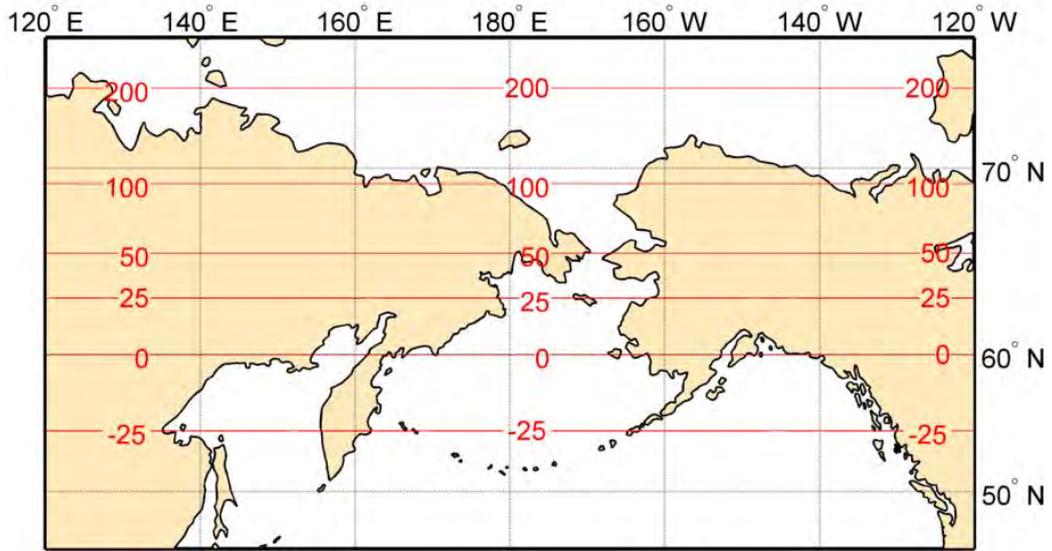
Figure 65. Mercator projection – Isolines of areal distortion (%) [0, 25, 50, 100, 200]
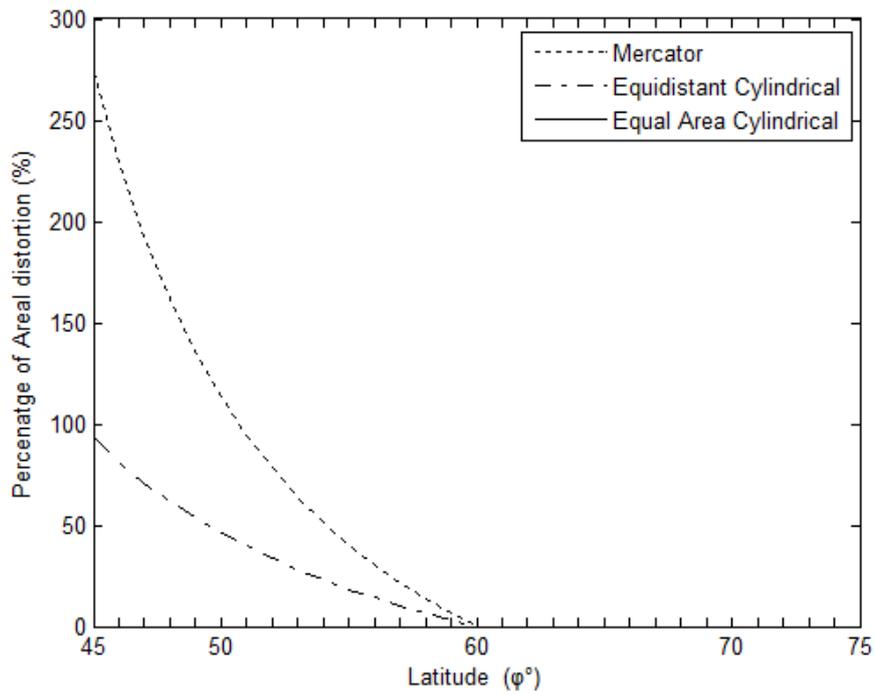
Figure 66. Cylindrical projections: Area scale factor
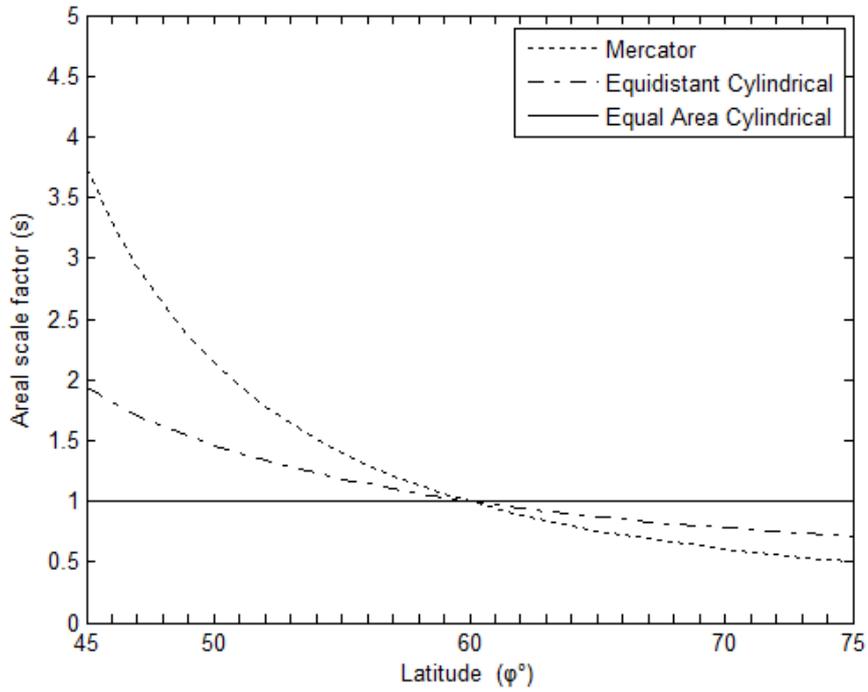
Figure 67. Cylindrical projections: Percentage of areal distortion
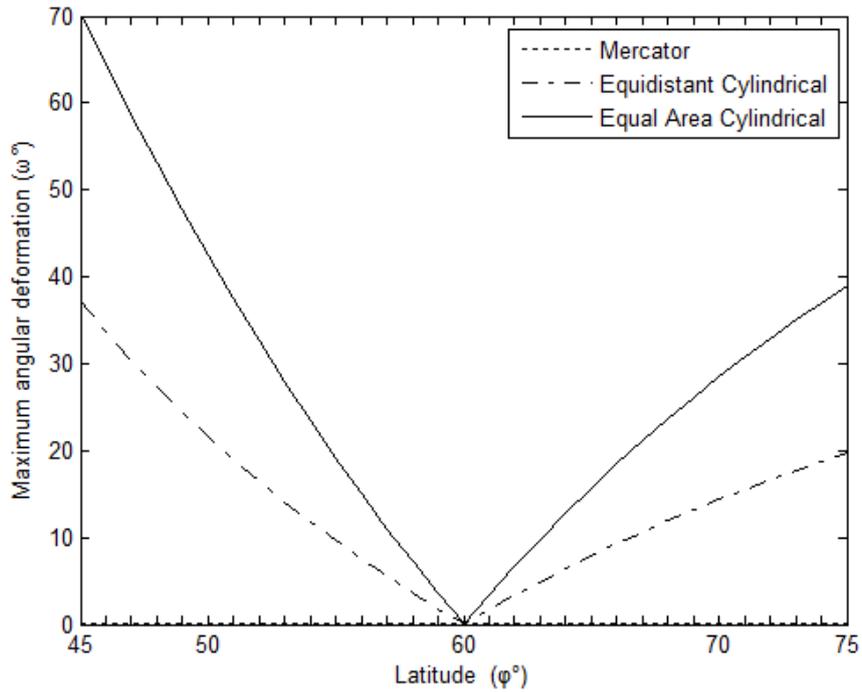
Figure 68. Cylindrical Projections: Maximum angular distortion (ω) in degrees
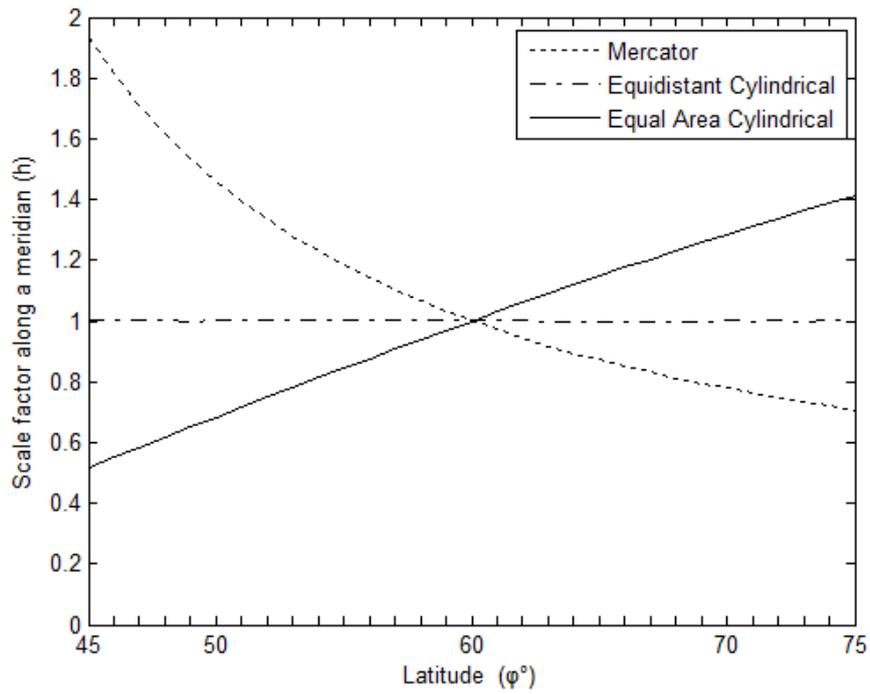
Figure 69. Cylindrical projections: Scale factor along a meridian (h)
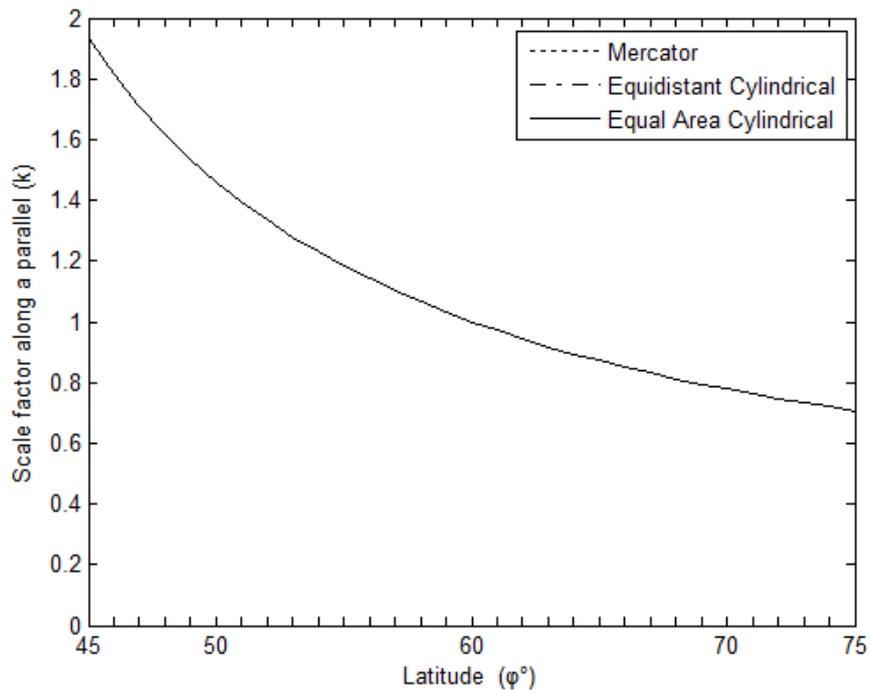
Figure 70. Cylindrical projections: Scale factor along a parallel (h)

	mercator (φo = 60)					equidistance (φo = 60)					equalarea (φo = 60)				
φ	area	area %	angles	k	h	area	area %	angles	k	h	area	area %	angles	k	h
75	3,75	274,62	0	1,94	1,94	1,94	93,64	37,01	1,93	1	1	0	70,47	1,93	0,52
74	3,30	230,32	0	1,82	1,82	1,82	81,83	33,57	1,82	1	1	0	64,48	1,81	0,55
73	2,94	193,61	0	1,71	1,71	1,71	71,43	30,33	1,71	1	1	0	58,68	1,71	0,59
72	2,63	162,85	0	1,62	1,62	1,62	62,20	27,26	1,62	1	1	0	53,08	1,62	0,62
71	2,37	136,82	0	1,54	1,54	1,54	53,96	24,34	1,54	1	1	0	47,67	1,54	0,65
70	2,15	114,60	0	1,46	1,46	1,47	46,56	21,58	1,46	1	1	0	42,46	1,46	0,68
69	1,95	95,49	0	1,40	1,40	1,40	39,88	18,95	1,40	1	1	0	37,43	1,39	0,72
68	1,79	78,92	0	1,34	1,34	1,34	33,82	16,44	1,34	1	1	0	32,59	1,33	0,75
67	1,64	64,47	0	1,28	1,28	1,28	28,30	14,05	1,28	1	1	0	27,94	1,28	0,78
66	1,52	51,79	0	1,23	1,23	1,23	23,26	11,77	1,23	1	1	0	23,46	1,23	0,81
65	1,41	40,61	0	1,19	1,19	1,19	18,63	9,58	1,18	1	1	0	19,15	1,18	0,85
64	1,31	30,70	0	1,14	1,14	1,14	14,37	7,49	1,14	1	1	0	15,01	1,14	0,88
63	1,22	21,87	0	1,10	1,10	1,10	10,44	5,49	1,10	1	1	0	11,03	1,10	0,91
62	1,14	13,98	0	1,07	1,07	1,07	6,81	3,58	1,07	1	1	0	7,20	1,06	0,94
61	1,07	6,89	0	1,03	1,03	1,03	3,43	1,74	1,03	1	1	0	3,53	1,03	0,97
60	1	0,50	0	1	1	1	0	0	1	1	1	0	0	1	1
59	0,95	-5,27	0	0,97	0,97	0,97	-2,63	1,72	0,97	1	1	0	3,39	0,97	1,03
58	0,89	-10,51	0	0,95	0,95	0,95	-5,36	3,35	0,94	1	1	0	6,64	0,94	1,06
57	0,85	-15,27	0	0,92	0,92	0,92	-7,91	4,91	0,92	1	1	0	9,77	0,92	1,09
56	0,80	-19,62	0	0,90	0,90	0,90	-10,30	6,42	0,90	1	1	0	12,77	0,89	1,12
55	0,76	-23,59	0	0,87	0,87	0,87	-12,55	7,87	0,87	1	1	0	15,65	0,87	1,15
54	0,73	-27,23	0	0,85	0,85	0,85	-14,66	9,26	0,85	1	1	0	18,42	0,85	1,18
53	0,69	-30,58	0	0,83	0,83	0,83	-16,64	10,61	0,83	1	1	0	21,07	0,83	1,20
52	0,66	-33,66	0	0,81	0,81	0,81	-18,52	11,90	0,81	1	1	0	23,62	0,81	1,23
51	0,64	-36,50	0	0,80	0,80	0,80	-20,28	13,14	0,80	1	1	0	26,07	0,79	1,26
50	0,61	-39,12	0	0,78	0,78	0,78	-21,95	14,35	0,78	1	1	0	28,42	0,78	1,28
49	0,58	-41,56	0	0,76	0,76	0,76	-23,52	15,50	0,76	1	1	0	30,68	0,76	1,31
48	0,56	-43,81	0	0,75	0,75	0,75	-25,01	16,62	0,75	1	1	0	32,85	0,75	1,34
47	0,54	-45,90	0	0,74	0,74	0,74	-26,42	17,69	0,73	1	1	0	34,93	0,73	1,36
46	0,52	-47,85	0	0,72	0,72	0,72	-27,76	18,73	0,72	1	1	0	36,93	0,72	1,39
45	0,50	-49,67	0	0,71	0,71	0,71	-29,03	19,73	0,71	1	1	0	38,85	0,71	1,41

Table 10. Cylindrical projections: Values for area distortion, Percentage of areal distortion, Maximum angular distortion (ω) in decimal degrees, Scale factor along a parallel (k), Scale factor along a meridian (h) for Mercator (standard parallel φ = 60º), Equidistant Cylindrical (standard parallel φ = 60º) and Equal Area Cylindrical (standard parallel φ = 60º).

2.5   Study of the shape of Great Circles/ Rhumb Lines and the shape of Graticule lines

The study of the shape of Great Circles and Rhumb Lines is carried out through the computation and portrayal of selected navigational paths in the Polar area on those projections, which satisfy the tolerance values set for the first criterion. These projections are:

- the Polar Equidistant (Figures 75, 76, 77 & 78) and the Polar Stereographic (Figures 73, 74 & 79) suitable mainly for the Arctic Region

- the Lambert Conformal (Figure 71) and the Conic Equidistant (Figure 72) suitable mainly for the sub Arctic Region

In the polar area the Orthodromic navigation is recommended due to the fact that the Orthodromic distance is shorter than the Loxodromic one. This difference becomes greater as the distance between the departure and destination points increases. In order to provide accurate results for the Orthodromic distances they are calculated as Great Elliptic arcs, which offer higher accuracy than that of the Great Circle arcs (Pallikaris and Latsas, 2009).

With respect to the shape of the graticule lines and more specifically of the representation of meridians and the way they are intersected with parallels, this criterion becomes less important due to the significance of the other two criteria and the fact that – as it has already pointed out – all three criteria cannot be satisfied simultaneously. In any case this last criterion will be taken into account in case it is partially satisfied.

As it is derived from both the distances computed and the shape of the Great Circles, all four projections considered as suitable due to their distortion characteristics, portray Great Circles as straight or near-straight lines and the computed distances of the Great Circles paths (Orhodroms) are much less compared to the corresponding Loxodromic paths.

In addition to the above in all four projections meridians are portrayed as straight lines.

In the following figures the Orhodromes are depicted in solid blue lines and the Loxodromes in dashed red lines. In order to provide a clearer picture of the situation a small and a large scale version of the area concerned is depicted.
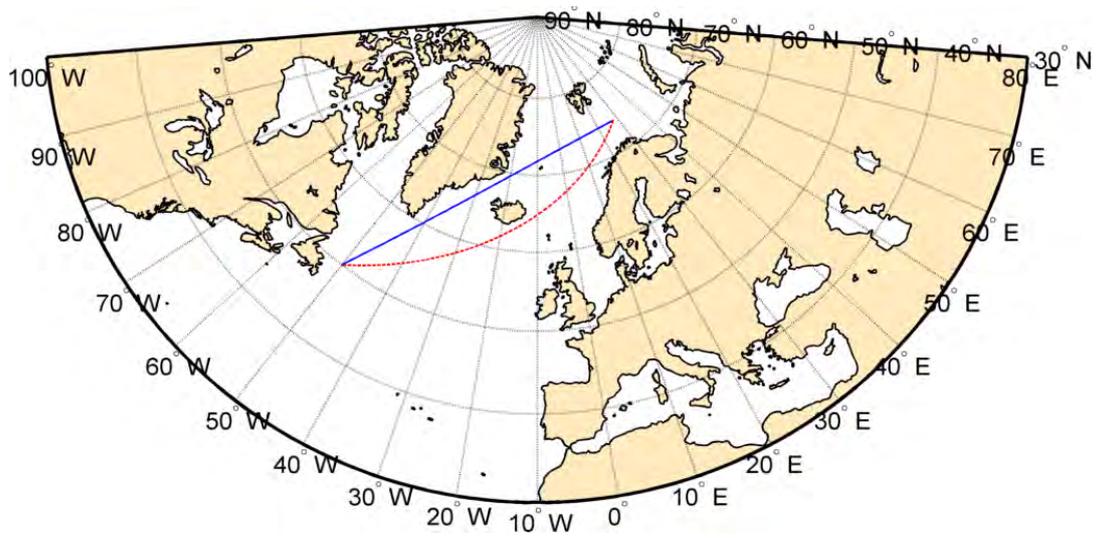
Figure 71. Lambert Conformal projection (standard parallels at latitudes 58 º and 82º), Departure point at 50º N, 50º W, destination point at 74º N, 28º E, Orthodromic distance 1550.1 n.m., Loxodromic distance 1557.5 n.m.
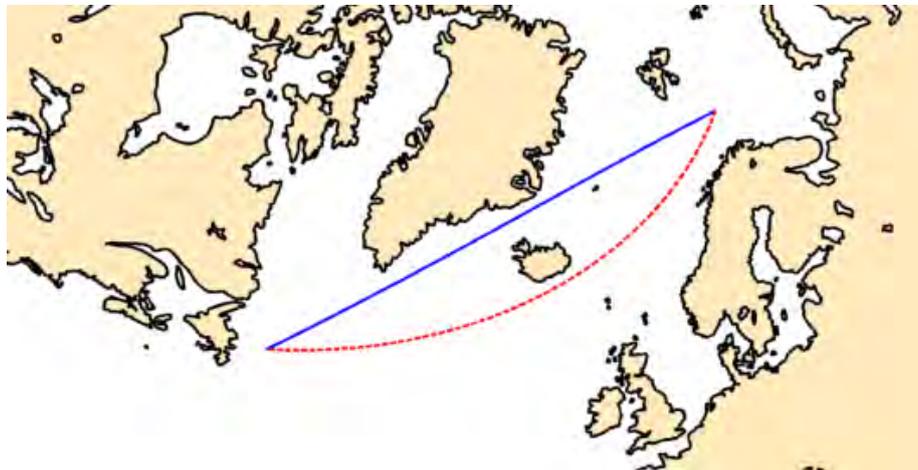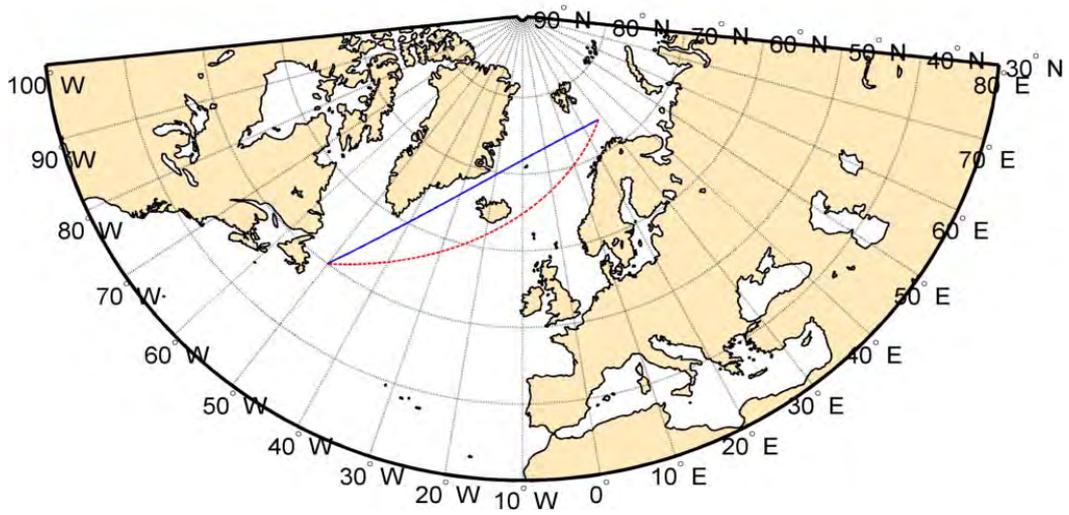
Figure 72. Conic Equidistant projection (standard parallels at latitudes 58 º and 82º), Departure point at 50º N, 50º W, destination point at 74º N, 28º E, Orthodromic distance 1550.1 n.m., Loxodromic distance 1557.5 n.m.
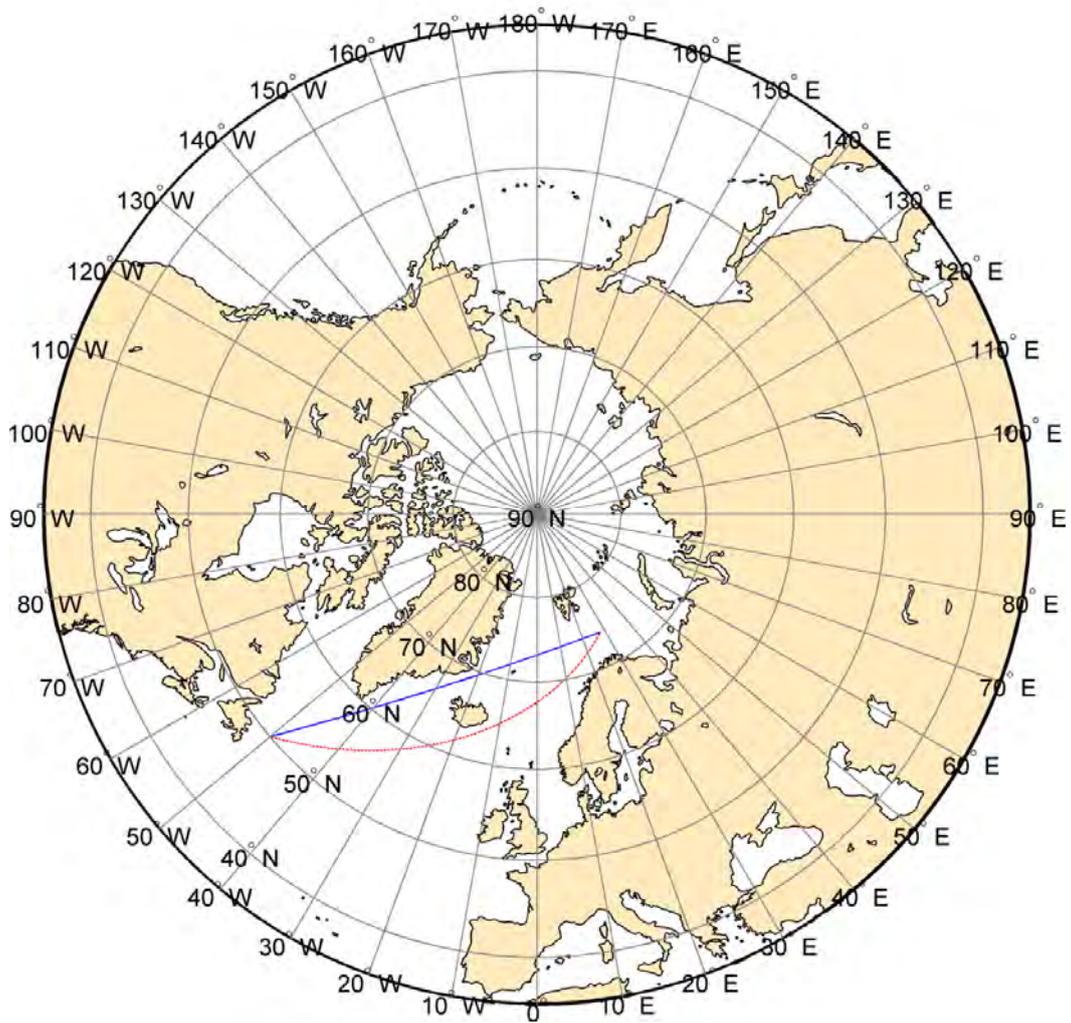
Figure 73. Azimuthal Polar Stereographic projection, Departure point at 50º N, 50º W, destination point at 74º N, 28º E, Orthodromic distance 1550.1 n.m., Loxodromic distance 1557.5 n.m.

Figure 74. Azimuthal Polar Stereographic projection, Departure point at 50º N, 50º W, Destination point at 74º N, 28º E, Orthodromic distance 1550.1 n.m, Loxodromic distance 1557.5 n.m.
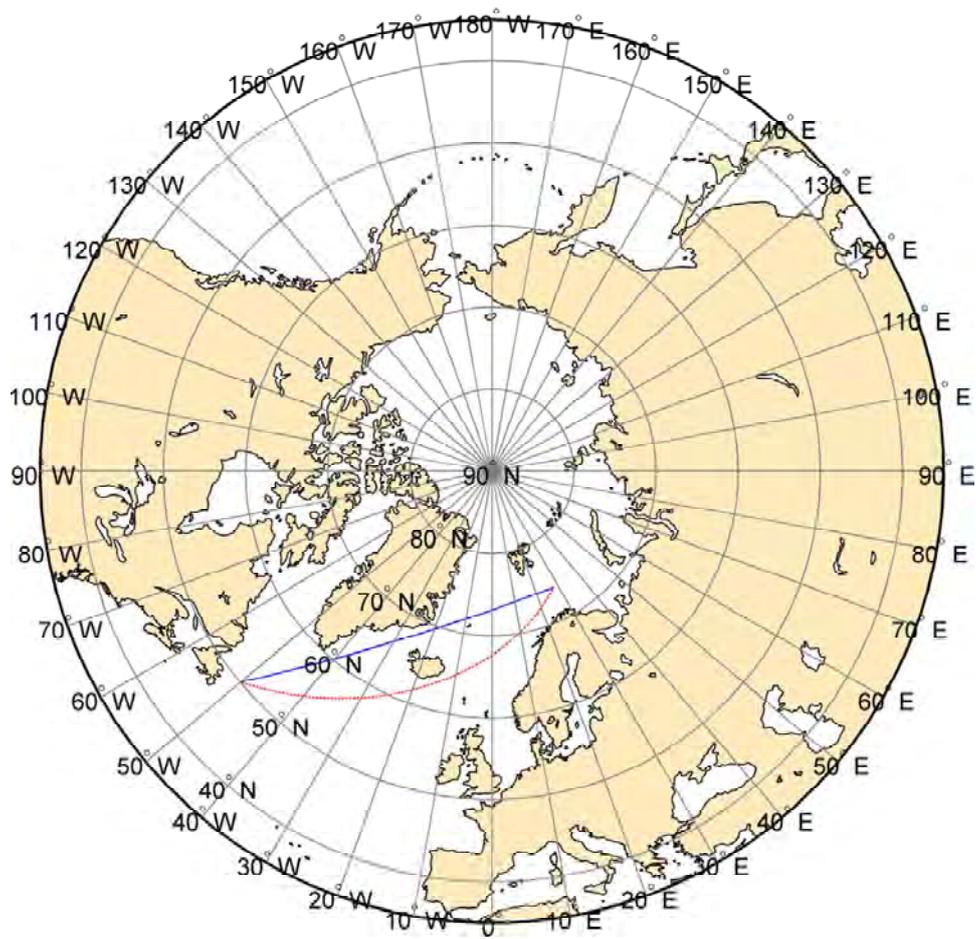
Figure 75. Azimuthal Polar Equidistant projection, Departure point at 50º N, 50º W, Destination point at 74º N, 28º E. Orthodromic distance 1550.1 n.m., Loxodromic distance 1557.5 n.m.

Figure 76. Azimuthal Polar Equidistant projection, Departure point at 50º N, 50º W, Destination point at 74º N, 28º E. Orthodromic distance 1550.1 n. m., Loxodromic distance 1557.5 n.m.

Figure 77 Azimuthal polar Equidistant projection, Departure point at 67º N 169º W, Destination point at 67º N 3º E, Central meridian at long 175º, Orthodromic distance 2765.6 n.m., Loxodromic distance 4051.1 n.m
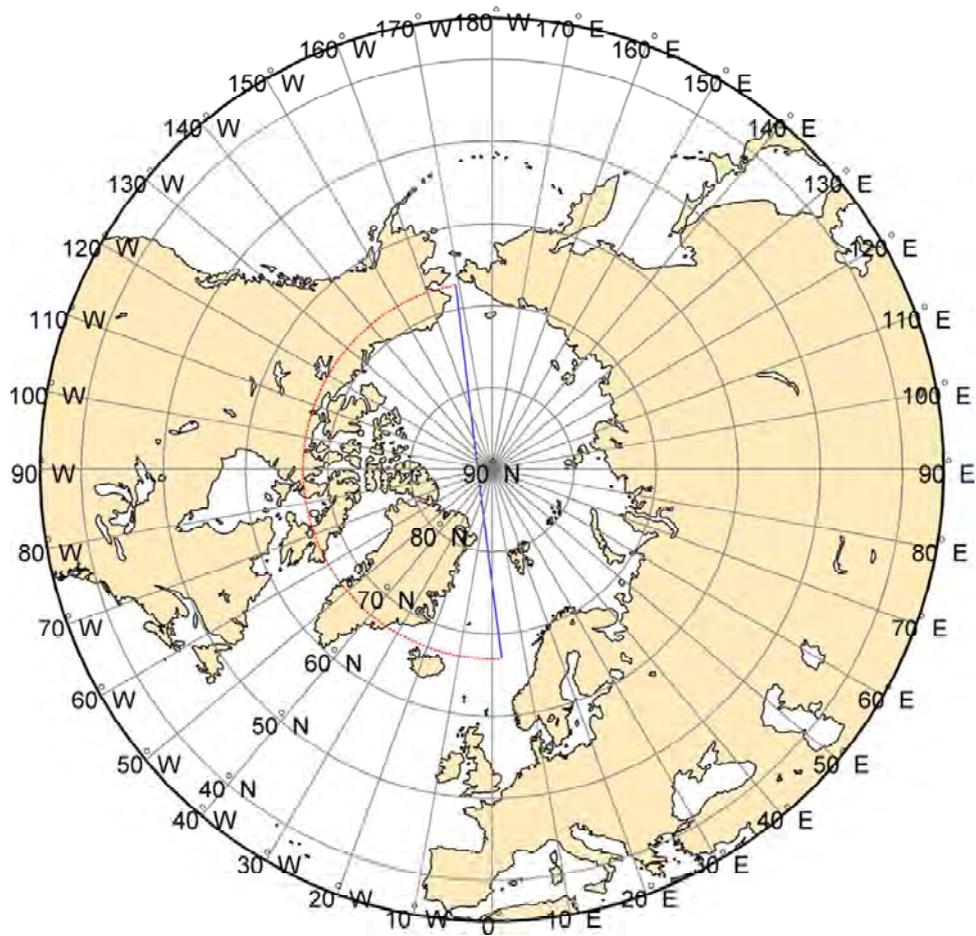
Figure 78. Azimuthal polar Equidistant projection, Departure point at 67º N 169º W, Destination point at 67º N 3º E, Central meridian at long 000º, Orthodromic distance 2765.6 n.m., Loxodromic distance 4051.1 n.m.
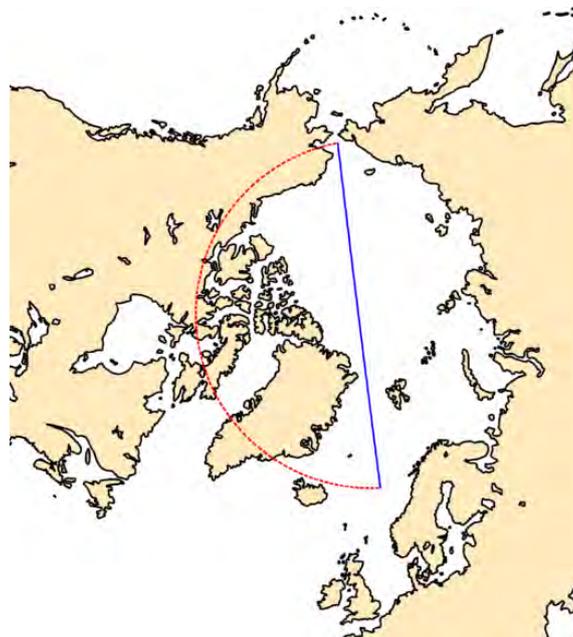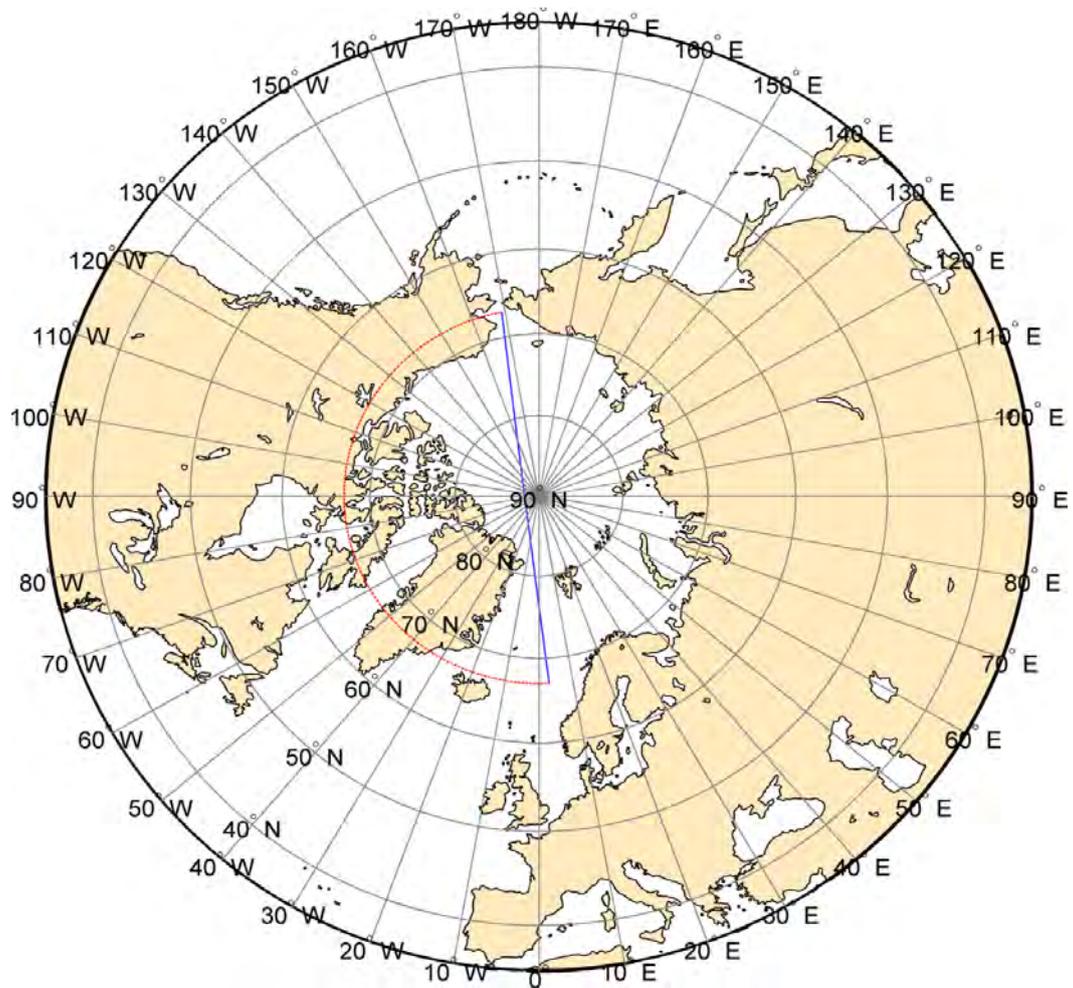
Figure 79. Azimuthal Stereographic projection, Departure point at 67º N 169º W, Destination point at 67º N 3º E, Central meridian at long 000º, Orthodromic distance 2765.6 n.m., Loxodromic distance 4051.1 n.m.

3   Conclusions

The decision on the choice of the suitable projection for a particular map/chart is a problem that cartographers are facing quite often. A number of factors influence such decision, which are relative to:

    a.   the shape and the location of the area covered,

    b.   the magnitude and distribution of distortions which are inherent to any cartographic projection,

    c.   specific characteristics which are relevant to the intended use of the map/chart

The primary aim of a logical choice is to select a projection in which "the extreme distortions are smaller than would occur in any other projection used to map the same area" [Maling, 1995]. This has been the main criterion for the selection of suitable projection for the Arctic Region in this study. Besides this criterion the purpose and the indented use of the map/chart determine which property/criterion is important.

Such an effort requires analytical approach for the calculation of the values of each kind of distortion for the projections under consideration. The computed distortions are subsequently recorded in tables and portrayed as ellipses of distortion, isolines of distortion etc., which reveal the suitability of a projection for the indented use. On the other hand the analytical approach allows for the identification of the geographical boundaries of the areas where the distortions' values satisfy the specified acceptable limits. In this project the aforementioned characteristic led to the "separation" of the study area in Arctic and sub-Arctic Regions for which different projections have been chosen and proposed.

More specifically, for the Arctic Region the **Azimuthal Polar Equidistant** projection and the **Azimuthal Polar Stereographic** projection are proposed. For the sub Arctic Region the **Lambert Conic Conformal** projection and the **Conic Equidistant** projection are considered as the most suitable ones. All four projections proposed can be used for both the traditional nautical chart and the ECDIS and are considered as the starting point for further study of specific ECDIS requirements (i.e. radar integration with ENC).

Given that most of the ECDIS systems do not include these projections either as a system or a manual choice, it is considered necessary that the specifications concerning the performance requirements of ECDIS be revised in order to include the proposed projections for the Arctic and sub Arctic Regions.

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5 Table of figures